Macroeconomic cycles and bond return predictability

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Abstract

Motivated by prior evidence that the price of risk varies across frequencies, we study the predictability of monthly excess bond returns estimating latent factors generating common macroeconomic cycles of different lengths. Our method combines a new *band spectrum principal component estimator* for frequency-specific factors and supervised learning. Not all macroeconomic cycles are found to predict bond returns in real time, on the contrary, predictability concentrates only at some bands of frequencies. Two factors are powerful out-of-sample predictors: an *inflation factor* obtained maximizing common macroeconomic cycles of at least 8 years and a *term spread factor* obtained maximizing common macroeconomic cycles of 1 to 3 years. We find that the inflation factor is unspanned by the current cross-section of yields (and forward rates) and relatively more accurate at shorter maturities and during recessions. The term spread factor is relatively more accurate at longer maturities and during expansions. Unlike all previous works using nonoverlapping returns and data available in real-time, our forecasts and generate sizeable economic value for investors of various kinds. In line with models based on countercyclical risk aversion, our factors generate countercyclical expected returns and term premia, and higher predictability during recessions.

JEL subject classification: C38, C53, C55, G11, G12, G17.
Key words: Bond return predictability, real-time macro data, frequency-specific factors, band-spectrum principal components, machine learning.

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1. Introduction

While the analysis of cyclical variation in the real activity has been one of the most essential problems in macroeconomics, it is just in the last few years that the study of cycles of different lenghts or, equivalently, decompositions across frequencies started to attract considerable interest in finance. Nonetheless, as of today a large body of evidence has already documented frequency-specific effects in asset prices. For example, Dew-Becker and Giglio (2016) show that once the risk of consumption fluctuations in asset pricing models is decomoposed in the frequency domain, long-run risk is robustly priced in the equity market. Also based on frequency domain techniques, Neuhierl and Varneskov (2021) study the contribution of transitory and persistent components of the stochastic discount factor to the unconditional asset return premium and find considerable evidence in favour of frequency-dependent risk in the US. A time-scale regression approach is instead adopted by Bandi et al. (2021) for "spectral betas" in cross-sectional asset pricing models, and Bandi et al. (2019) for stock return predictability. While these are just some of the most notable contributions to a rapidly growing literature, our paper extends the study of frequency-specific effects to the case of bond return predictability and, following an abundant literature of macroeconomic factors in bond risk premia, proposes a new method for the estimation of latent factors driving common macroeconomic cycles of given lengths.[1]

According to the expectations hypothesis (EH) of the term structure of interest rates the long-term rate is equal to the average of expected future short rates plus a constant risk premium. While results against the EH span the last four decades (Fama and Bliss, 1987; Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005), the driving forces of time-variation in bond risk premia are still intensely debated. Similarly to the case of the equity premium studied by Welch and Goyal (2008), part of this empirical debate owes to the difficulties in obtaining more accurate out-of-sample excess bond return predictions than the historical average benchmark implied by the EH. If investors demand compensation for the risk of recessions as in notable rational expectations models (Campbell and Cochrane, 1999; Wachter, 2006), excess bond returns should be predictable and evolve with the expected macroeconomic conditions. Resorting to dynamic factor analysis Ludvigson and Ng (2009) show that, unlike observed predictors, latent common macroeconomic factors estimated via principal components from a large macroeconomic dataset, contain significant out-of-sample predictive information unspanned by the current yield curve and expected bond returns are consistent with countercyclical risk aversion. Similarly, other influential works such as Cooper and Priestley (2009), Greenwood and Vayanos (2014), Joslin et al. (2014) and Cieslak and Povala (2015) established a link between the state of the economy and bond return predictability. More recently, however, Ghysels et al. (2018) find that once real-time data is considered the predictive power of latent macroeconomic factors like those considered by Ludvigson and Ng (2009) vanishes.[2]

[1] See also Ortu et al. (2013), Kamara et al. (2016), Chaudhuri and Lo (2018), Bandi and Tamoni (2023), Bandi and Su (2023) among many others.

[2] See, however, Caruso and Coroneo (2023) who show that when the latest data vintages available in real time are used, instead of first releases as in Ghysels et al. (2018), predictions of interest rates are nearly as accurate as

As of today, a burgeoning literature has adopted more sophisticated machine learning methods. Motivated by the possible existence of irrelevant variation (unrelated to future bond returns) in the predictors and/or nonlinearities of unknown form, increasing interest has been devoted to supervised learning (see Bianchi et al., 2021; Huang et al., 2023; Huang and Shi, 2023, among others). Even when real-time data is used, the evidence of predictability in these works still comes with at least one of the following two limitations. First, the adoption of overlapping returns which imply an annual holding period. This choice has been criticized since with annual holding periods important short-run dynamics — such as Lehman Brothers' bankruptcy — and business cycle turning points are overlooked (Gargano et al., 2019; Wan et al., 2022). Fan et al. (2022) show that this is far from being an innocuous choice: evidence of predictability in overlapping returns produced by deep learning approaches becomes weak in nonoverlapping returns. Second, there is a difficulty in translating statistical forecasting accuracy into economic value for investors. For example, in the work of Wan et al. (2022) forecasts which are more accurate in mean square error terms than the historical average benchmark are often associated with poor portfolio performance. First raised by Thornton and Valente (2012) and Sarno et al. (2016), this is still an open issue, especially as far as real-time nonoverlapping returns forecasting is concerned. Indeed, to the best of our knowledge, no predictive method considered thus far has been found to generate any economic value in real-time using nonoverlapping excess bond returns. Significant certain equivalent return (CER) gains are found by Eriksen (2017), Bianchi et al. (2021), Huang et al. (2023) using overlapping returns, and by Gargano et al. (2019) using nonoverlapping returns and fully revised macroeconomic data.[3]

While our work follows the recent trend of machine learning methods by allowing for nonlinearity and a high-dimensional space of predictors, our framework is in the tradition of the seminal work of Ludvigson and Ng (2009) because we consider latent common macroeconomic factors and a dynamic factor model. Inspired by a number of recent works finding frequency-specific effects in financial data, we extend this framework by allowing latent macroeconomic factors to be frequency-specific in the sense that they generate common macroeconomic cycles of given lengths. Our *band spectrum factor model* is nonlinear since in its frequency-domain representation factor loadings are allowed to change across bands of frequencies. At the same time, it has a linear time-domain representation with frequency-specific factors.

We show that common factors affecting a band of frequencies can be estimated via a generalized principal component estimator which is obtained by taking the component of the covariance matrix associated with the same band of frequencies. As a result, we estimate frequency-specific factors by maximizing specific cyclical comovements of the variables, rather than the comovements associated with common cycles of all lenghts. In analogy with Engle (1974) who considers the same setup but with observed factors (known as band spectrum regressions), we refer to our estimator as *Band Spectrum Principal Components* (BSPCs). The principal component estima-

those obtained with fully revised macroeconomic data.

[3] Huang et al. (2023) and the corrigendum to Bianchi et al. (2021) find statistical evidence of predictability based on nonoverlapping returns, but they perform no economic evaluation.

tor emerges as a limiting case when macroeconomic cycles of all lengths (i.e. the full spectrum) are considered.

Do common macroeconomic cycles of all lengths predict excess nonoverlapping bond returns? In order to answer this question we first need to detect the latent macroeconomic factors related to expected bond returns. In fact, Ludvigson and Ng (2009) show that for predicting bond returns it is necessary to disentangle relevant from irrelevant macroeconomic factors, and identify a subset of factors with predictive power via an extensive model selection procedure based on the minimization of a BIC criterion. Similarly to Huang et al. (2023), we do so by adopting a supervised learning approach based on the principle of statistical sufficiency: rather than searching for a subset of factors which predicts bond returns, we focus on the space they span. This space is identified by projecting each predictor onto observable proxies before extracting principal components (Fan et al., 2017, 2021). Similarly to identification methods via instrumental variables, these proxies fulfill exogeneity since they are orthogonal to common macroeconomic factors unrelated to future bond returns.[4] Our procedure is the same but we extract BSPCs. That is, our predictors are factors exctracted by choosing proxies for the central subspace and a band of frequencies. In so doing, we are able to study whether excess bond returns live in a subspace of common macroeconomic cycles of given lengths.

From a real-time macroeconomic dataset of 54 variables, we extract two frequency-specific factors across different bands of frequencies, one using inflation as a proxy, the other the term spread. This is in line with a broad literature combining yield and possibly unspanned macroeconomic factors. While weak or no evidence of predictability is found when full-spectrum predictors are considered, the picture is remarkably different when we instead focus on specific bands of frequencies. Two powerful predictors are obtained within two different spectral bands: the one by taking factors driving macroeconomic cycles of at least 8 years related to the inflation, the other macroeonomic cycles of 1 to 3 years related to the term spread. Using these two predictors — hereafter referred to as *inflation factor* and *term spread factor* — we find evidence of predictability in both statistical and, most important, economic terms for investors of various kinds (i.e. with mean-variance or power utility and a range of risk aversions). In fact, to the best of our knowledge, the finding of significant CER gains using real-time data and nonoverlapping returns is novel in this literature. Our inflation factor contains macroeconomic information which is unspanned by the usual yield curve factors (level, slope and curvature) or forward rates (Cochrane and Piazzesi (2005)'s factor) and is relatively more accurate for shorter maturities and during recessions. The term spread factor is instead more accurate for longer maturities and during expansions. The forecasts produced by these two predictors are in line with the dominant view that risk premia are countercyclical. We conclude so by observing that they generate: expected returns which are negatively correlated with cyclical indicators (especially, the Michigan consumer sentiment index), countercyclical term premia, higher statistical accuracy and larger economic value during recessions.

[4] See also Kelly et al. (2019) for a similar method for the cross-section of returns based on unobserved factors with loadings related to observable instruments.

Being our analysis based on real-time data, nonoverlapping returns, and forecasts with economic value for investors, we reaffirm three main conclusions of Ludvigson and Ng (2009) without the above two limitations in recent works. First, we confirm the existence of comovements between the macroeconomy and predictable variation in excess bond returns which are captured by some subset of latent macroeconomic factors. Second, we reject the expectations hypothesis in favour of countercyclical risk aversion. Third, we add to the evidence against the "spanning hypothesis" and suggests that affine term structure models should include macroeconomic information which is not spanned by current the cross-section of yields (among others, Chernov and Mueller (2012), Joslin et al. (2014), Coroneo et al. (2016) also associate unspanned macroeconomic factors with inflation).

At the same time, by allowing for frequency-specific factors, our analysis of macroeconomic cycles of different lengths gives a more detailed picture of bond return predictability than the one emerging from previous works based on latent macroeconomic factors. This enables us to note that our results are in line with other findings in the literature. Our inflation factor is consistent with long-run risk models such as that of Bansal and Shaliastovich (2013) who establish a link between long-run inflation expectations and bond premia.[5] Our term spread factor confirms the findings of Fama and French (1989) who conclude that the "term spread is more closely related to the shorter-term business cycles identified by NBER", and resembles the cycle factor identified by Cieslak and Povala (2015), that is, a component of yields which is orthogonal to the trend inflation and whose predictive power increases across maturities. Furthermore, the results on our term spread factor bear some similarities to those of Andreasen et al. (2021) who, like us, find that term structure variables are powerful predictors during expansions.

The rest of this paper is as follows. In Section 2 we outline all the methodological aspects of our work: our band spectrum factor model, its estimation, and the supervised learning method we adopt to obtain predictors for our forecasting exercise. Section 3 is dedicated to the yield data and the real-time macroeconomic dataset used to construct our predictors. Out-of-sample forecasting results are presented in Section 4. The links to the real economy and the implications for rational expectations models are explored in Section 5. Section 6 concludes.

2. Methodology

There are two difficulties associated with the widespread use of principal components in predictive regressions for bond returns (typically common factors estimated using large macroeconomic datasets).

First, being linear combinations with maximum variance, principal components account for variables' comovements at all frequencies by aggregating cycles of all lengths. Albeit this is adequate for predicting processes of various kinds, mounting evidence that systematic risk

[5]However, at odds with our result that the inflation factor is unspanned, in Bansal and Shaliastovich (2013) the spanning hyphothesis holds true.

varies across frequencies motivates us to investigate whether macroeconomic cycles of different lengths have the same relationship, or any relationship whatsoever, with bond returns. If some macroeconomic cycles contain no predictive power for excess bond returns, the exclusion of the corresponding frequencies reduces the measurement error in the predictors. This means that accurate predictors cannot be obtained aggreragating cycles of all lengths. Similarly, if cycles of different lengths do not have the same relationship with bond returns, frequency-specific predictive systems should be allowed for. In these cases, principal components become suboptimal predictors.

In order to shed light on the possible existence of frequency-specific predictors, we develop an approach to account for comovements among cycles of given lengths. In Section 2.1, we introduce a novel factor model with frequency-specific factors. The model arises as a natural consequence of variation in the factor loadings across spectral bands, which we allow for following the seminal work of Engle (1974) on band-spectrum regressions. In Section 2.2, we propose an estimator for frequency-specific factors. Our *band spectrum principal components* are linear combinations of variables with maximum variance only within a band of frequencies, hence they generalize principal components.

Second, not all macroeconomic comovements need to contain information on future bond returns. In fact, Ludvigson and Ng (2009) found that only a subset of common macroeconomic factors predict bond returns. In Section 2.3 we combine band spectrum principal components with supervised learning so that we allow our predictors to live in a subspace of frequency-specific common factors. In so doing, our predictors are not contamined by common macroeconomic cycles unrelated to predictable variation in excess bond returns.

2.1. Frequency-specific factors

Consider a $T \times N$ panel $\mathbf{X} := \{x_{it};\ i = 1, \ldots, N;\ t = 1, \ldots, T\}$ of mean-zero weakly stationary variables with a latent factor structure

$$\mathbf{X}_t = \Lambda \mathbf{F}_t + \mathbf{e}_t \tag{1}$$

where $\mathbf{X}_t$ is the N-dimensional vector $(x_{1t}, x_{2t}, \ldots, x_{Nt})'$, Λ a $N \times r$ matrix of loadings, $\mathbf{F}_t$ an r-dimensional vector of unobservable factors, $\mathbf{e}_t$ a N-dimensional vector of idiosyncratic terms which are weakly cross-correlated in the sense of Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986), and orthogonal to $\mathbf{F}_t$ at all leads and lags.[6] Being the factors common to all cross-sectional units $x_{1t}, \cdots, x_{Nt}$, the term $\Lambda \mathbf{F}_t$ is known as the common component of $\mathbf{X}_t$ and interpreted as the effect of comovements between the variables. In this

[6] Following Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986), we consider an approximate factor structure for which the cross-correlation generated by the idiosyncratic components is asymptotically negligible. Exact factor structures assume instead that $\mathbf{e}_t$ has a diagonal covariance matrix. Orthogonality at all leads and lags between the factors and idiosyncratic terms is assumed for simplicity. It could be relaxed to allow for some weak dependence as in Assumption D of Bai and Ng (2002).

work we focus on a frequency-specific analysis of those comovements. Letting $\iota = \sqrt{-1}$ be the imaginary unit, ω some frequency in $[-\pi, \pi]$, we have the Fourier transforms $\mathcal{X}_\omega = \sum_{t=1}^{T} \mathbf{X}_t e^{-\iota\omega t}$, $\mathcal{F}_\omega = \sum_{t=1}^{T} \mathbf{F}_t e^{-\iota\omega t}$, $\mathcal{E}_\omega = \sum_{t=1}^{T} \mathbf{e}_t e^{-\iota\omega t}$.[7] The factor model (1) allows for a frequency-domain representation

$$\mathcal{X}_\omega = \Lambda \mathcal{F}_\omega + \mathcal{E}_\omega$$

which shows that the relationship between the cycles of length $2\pi/\omega$ of $\mathbf{X}_t$ and those of the same length of the common factors $\mathbf{F}_t$ is constant and independent of ω.

We are interested in a more general framework in which comovements are allowed to vary across frequencies. Consider, for example, a partition of $[-\pi, \pi]$ into two disjoint subsets Ω_1 and Ω_2.[8] Allowing for different cyclical comovements across these two bands of frequencies calls for a frequency-domain representation

$$\mathcal{X}_\omega = \begin{cases} \Lambda_1 \mathcal{F}_\omega + \mathcal{E}_\omega & \omega \in \Omega_1 \\ \Lambda_2 \mathcal{F}_\omega + \mathcal{E}_\omega & \omega \in \Omega_2 \end{cases} \tag{2}$$

where, similarly to Engle (1974), coefficients (factor loadings) are allowed to vary across frequencies. Equation (2) can be rewritten as

$$\mathcal{X}_\omega = \Lambda_1 \mathcal{F}_{\omega,1} + \Lambda_2 \mathcal{F}_{\omega,2} + \mathcal{E}_\omega \tag{3}$$

where $$\mathcal{F}_{\omega,1} = \begin{cases} \mathcal{F}_\omega & \omega \in \Omega_1 \\ \mathbf{0} & \omega \in \Omega_2 \end{cases} \quad \text{and} \quad \mathcal{F}_{\omega,2} = \begin{cases} \mathbf{0} & \omega \in \Omega_1 \\ \mathcal{F}_\omega & \omega \in \Omega_2 \end{cases}$$

As a result, we are interested in the *band spectrum factor model*

$$\mathbf{X}_t = \Lambda_1 \mathbf{F}_t(\Omega_1) + \Lambda_2 \mathbf{F}_t(\Omega_2) + \mathbf{e}_t \tag{4}$$

where $\mathbf{F}_t(\Omega_1)$, $\mathbf{F}_t(\Omega_2)$, defined as the inverse Fourier transforms of $\mathcal{F}_{\omega,1}$ and $\mathcal{F}_{\omega,2}$, are common factors across the spectral components of $\mathbf{X}_t$ at frequencies ω in Ω_1 and Ω_2, respectively.[9] Being the factors $\mathbf{F}_t(\Omega)$ unrelated to any frequency out of the band Ω, we refer to them as frequency-specific factors which generate the common cycles of $\mathbf{X}_t$ of length $2\pi/\omega$ for all frequencies $\omega \in \Omega$. So, to the nonlinear frequency-domain representation (2) corresponds a factor model (4) which is linear in the frequency-specific factors.

The band spectrum factor model (4) implies a canonical decomposition of the covariance

[7] In practice we consider the Fourier frequencies $\omega_k = \pi k/T$ with $k = -T, -T+1, \cdots, T$.

[8] For simplicity and without loss of generality, two bands of frequencies are considered in this section. In the empirical part of this work we consider four bands.

[9] The derivation of $\mathbf{F}_t(\Omega)$ corresponds to the frequency-domain band-pass filter (Priestley, 1981, p. 274–275). Albeit not as popular as its approximate time-domain counterpart, also this method has been applied to the to the measurement of business cycles (see Englund et al., 1992; Hassler et al., 1994).

matrix

$$\mathbf{C}_0 \equiv E\left(\mathbf{X}_t\mathbf{X}_t'\right) = \Lambda_1 E\left(\mathbf{F}_t\left(\Omega_1\right)\mathbf{F}_t\left(\Omega_1\right)'\right)\Lambda_1' + \Lambda_2 E\left(\mathbf{F}_t\left(\Omega_2\right)\mathbf{F}_t\left(\Omega_2\right)'\right)\Lambda_2' + E\left(\mathbf{e}_t\mathbf{e}_t'\right) \tag{5}$$

for which the first term is the covariance of the comovements at frequencies in Ω_1, the second is the covariance of the comovements at frequencies in Ω_2, and the last one is the "weak" (i.e. asymptotically negligible) covariance generated by idiosyncratic cycles of all lengths.[10]

In order to estimate the frequency-specific factors $\mathbf{F}_t(\Omega)$, one needs to disentangle common from idiosyncratic covariances in Ω. Of course, this is only possible with a prior estimate of the overall (common and idiosyncratic) comovements in Ω. Exploiting the inverse Fourier transform $\mathbf{C}_0 = \int_{-\pi}^{\pi}\mathbf{S}(\omega)\ d\omega$, where $\mathbf{S}(\omega) = (2\pi)^{-1}\sum_{k=-\infty}^{\infty} e^{-\iota k\omega}\mathbf{C}_k$ is the spectral density matrix at frequency ω and $\mathbf{C}_k = E\left(\mathbf{X}_t\mathbf{X}_{t-k}'\right)$, the component of $\mathbf{C}_0$ due to the covariance among all common and idiosyncratic cycles in Ω is

$$\mathbf{C}_0(\Omega) := \int_{\omega\in\Omega} E\left(\mathcal{X}_\omega\mathcal{X}_\omega'\right) d\omega = \int_{\omega\in\Omega}\mathbf{S}(\omega)\, d\omega \tag{6}$$

In the rest of this paper we refer to $\mathbf{C}_0(\Omega)$ as the *band spectrum covariance* matrix of $\mathbf{X}_t$ in Ω.

2.2. Band Spectrum Principal components

The estimation of common factors via asymptotic principal components is widespread and well-known (Bai and Ng, 2002; Stock and Watson, 2002a; Forni et al., 2000). Assuming that $T^{-1}\sum_{t=1}^{T}\mathbf{F}_t\mathbf{F}_t'$ and $N^{-1}\Lambda\Lambda'$ converge to some positive definite matrices (with distinct eigenvalues) as T and N grow to infinity is enough to ensure that r eigenvalues of $\Lambda T^{-1}\sum_{t=1}^{T}\mathbf{F}_t\mathbf{F}_t'\Lambda'$ diverge as N grows to infinity. This implies that r eigenvalues of the covariance matrix of the data $\mathbf{C}_0$ diverge as well. Further assuming that the covariance matrix of the idiosyncratic terms has bounded eigenvalues as N grows to infinity, and some moment conditions, the space spanned by the factors can be estimated from the eigenvalue-eigenvector decomposition of the sample covariance matrix of the data.

The covariance structure (5) generated by the band spectrum factor model (4) suggests that frequency-specific factors $\mathbf{F}_t(\Omega_1)$ and $\mathbf{F}_t(\Omega_2)$ can be estimated following the same logic within

[10] To see this, it is enough to note that

$$\begin{aligned}\mathbf{C}_0 = \int_{-\pi}^{\pi} E\left(\mathcal{X}_\omega\mathcal{X}_\omega'\right)\ d\omega &= \Lambda_1\int_{-\pi}^{\pi} E\left(\mathcal{F}_{\omega,1}\mathcal{F}_{\omega,1}'\right)\ d\omega\ \Lambda_1' \ +\ \Lambda_2\int_{-\pi}^{\pi} E\left(\mathcal{F}_{\omega,2}\mathcal{F}_{\omega,2}'\right)\ d\omega\ \Lambda_2' \ + E\left(\mathbf{e}_t\mathbf{e}_t'\right)\\ &= \Lambda_1\int_{\omega\in\Omega_1} E\left(\mathcal{F}_\omega\mathcal{F}_\omega'\right)\ d\omega\ \Lambda_1' \ +\ \Lambda_2\int_{\omega\in\Omega_2} E\left(\mathcal{F}_\omega\mathcal{F}_\omega'\right)\ d\omega\ \Lambda_2' \ + E\left(\mathbf{e}_t\mathbf{e}_t'\right)\end{aligned}$$

since, $\int_{\omega\in\Omega_2} E\left(\mathcal{F}_{\omega,1}\mathcal{F}_{\omega,1}'\right)\ d\omega = \mathbf{0}$, $\int_{\omega\in\Omega_1} E\left(\mathcal{F}_{\omega,2}\mathcal{F}_{\omega,2}'\right)\ d\omega = \mathbf{0}$, and $\mathbf{e}_t$ is orthogonal to all common factors.

a band of frequencies. Consider the covariance in the band Ω_1

$$\begin{aligned}\mathbf{C}_0(\Omega_1) = \int_{\omega\in\Omega_1} E\left(\mathcal{X}_\omega \mathcal{X}_\omega'\right) &= \Lambda_1 \int_{\omega\in\Omega_1} E\left(\mathcal{F}_{\omega,1}\mathcal{F}_{\omega,1}'\right)\, d\omega\ \Lambda_1' + \int_{\omega\in\Omega_1} E\left(\mathcal{E}_\omega\mathcal{E}_\omega'\right)\, d\omega \\ &= \Lambda_1 E\left(\mathbf{F}_t(\Omega_1)\,\mathbf{F}_t(\Omega_1)'\right)\Lambda_1' + \int_{\omega\in\Omega_1} E\left(\mathcal{E}_\omega\mathcal{E}_\omega'\right)\, d\omega \qquad (7)\end{aligned}$$

where we used equation (3) and (4). Assuming that $N^{-1}\Lambda_1\Lambda_1'$ and $T^{-1}\sum_{t=1}^{T}\mathbf{F}_t(\Omega_1)\,\mathbf{F}_t(\Omega_1)'$ converge to positive definite matrices (with distinct eigenvalues) as $N\to\infty$ and $T\to\infty$ respectively, we have that r eigenvalues of $\mathbf{C}_0(\Omega_1)$ diverge as $N\to\infty$. This, combined with the usual assumptions on the idiosyncratic errors mentioned above, implies that as N, T jointly grow to infinity, the second term of (7) becomes negligible and the eigenvectors associated with largest r eigenvalues of $\mathbf{C}_0(\Omega_1)$ span the space of $\mathbf{F}_t(\Omega_1)$. This motivates the *band spectrum principal component* estimator

$$\hat{\mathbf{F}}_t(\Omega_1) = \sqrt{T}V_r'(\Omega_1)\,\mathbf{X}_t \qquad (8)$$

where $V_r(\Omega_1) = (v_1(\Omega_1), v_2(\Omega_1), \ldots\ v_r(\Omega_1))$ and $v_j(\Omega_1)$ is the eigenvector associated with the j-th largest eigenvalue of $\hat{\mathbf{C}}_0(\Omega_1)$ for $j\leq r$. Similarly, $\hat{\mathbf{F}}_t(\Omega_2) = \sqrt{T}V_r'(\Omega_2)\,\mathbf{X}_t$.

The band spectrum covariance $\mathbf{C}_0(\Omega)$ can be estimated by replacing $\mathbf{S}(\omega)$ in equation (6) with its estimate. We use the lag-window estimator

$$\hat{\mathbf{S}}(\omega) = \sum_{j=-M_T}^{M_T} K_j(M_T)\, e^{-\iota j\omega}\hat{\mathbf{C}}_j \qquad (9)$$

where $\hat{\mathbf{C}}_j$ is the sample estimate of $\mathbf{C}_j$, and $K_j(M_T) = 1 - \frac{|j|}{M_T}$ is the triangular kernel with bandwidth M_T, which is known to be consistent if $T^{-1}M_T\to 0$ as $T\to\infty$ and $M_T\to\infty$.[11] In practice $M_T = \lfloor\sqrt{T}\rfloor$ is often chosen, where $\lfloor\cdot\rfloor$ denotes the floor function. In the rest of this paper we refer to such estimator as

$$\hat{\mathbf{C}}_0(\Omega) = \int_{\omega\in\Omega}\hat{\mathbf{S}}(\omega)\, d\omega$$

While a formal proof on the consistent estimation of frequency-specific factors based on the above discussion is left to Appendix A, in Section 2.2.2 we provide simulation evidence that the proposed estimator performs well in finite samples.

2.2.1. Discussion

There are important antecedents to our band spectrum principal component estimator. The idea of estimating models on a band of frequencies dates back to the regression analysis with distributed lags of Hannan (1963, 1965). The most direct antecedent is however the seminal work on band spectrum regressions of Engle (1974) who, interested in studying whether slope coefficients change across frequencies, considers a usual least squares framework limited

[11] See Wu and Zaffaroni (2018).

to a band of frequencies. Although ours is a high-dimensional problem with a set of predictors driven by unobserved factors, our band spectrum principal component estimator is closely related to band spectrum regressions since it minimizes the least square objective function $\int_{\omega\in\Omega}(\mathcal{X}_\omega - \Lambda\mathcal{F}_\omega)'(\mathcal{X}_\omega - \Lambda\mathcal{F}_\omega)\ d\omega$. Indeed, this problem reverts to:

- OLS when $\Omega = [-\pi, \pi]$ and $\mathbf{F}_t$ is observed;
- principal components when $\Omega = [-\pi, \pi]$ and $\mathbf{F}_t$ is unobserved;
- Engle's band spectrum regressions when Ω is a subset of $[-\pi, \pi]$ and $\mathbf{F}_t$ is observed.

Our band spectrum principal component problem corresponds to the case in which Ω is a subset of $[-\pi, \pi]$ and $\mathbf{F}_t$ is unobserved.

Being the goal of band spectrum analysis the detection of possible frequency-specific effects, an essential property of these estimators is that when no frequency-specific effect holds the estimators yield inefficient estimates as compared to their full spectrum counterparts. For example, if the data generating process is a standard linear regression model band spectrum regressions across different bands yield inefficient estimates of the same slope coefficients which are constant across frequencies. Our BSPC estimator obeys the corresponding property when the loadings do not vary across bands. Returning to the band spectrum model (4), the BSPC estimator is such that $\hat{\mathbf{F}}(\Omega_j) = \mathbf{X}\hat{\Lambda}_j\left(\hat{\Lambda}_j\hat{\Lambda}_j'\right)^{-1}$, so the same estimate across the two bands is obtained when $\Lambda_1 = \Lambda_2$ and loadings are consistently estimated (see Appendix A). Intuitively, the BSPC becomes a relatively inefficient but consistent estimator of $\mathbf{F}$ since it only uses a band of frequencies even if there is no such thing as frequency-specific factor that can be obtained from the frequency-domain representation (2) when loadings are constant over the spectrum.

Note that even in absence of frequency-specific factors $\mathbf{F}(\Omega_j)$ remains well-defined as it corresponds to the band-pass filtered version of $\mathbf{F}$ in Ω_j. For this reason, in the simulation exercises of Section 2.2.2 we measure both the distance between $\hat{\mathbf{F}}(\Omega_j)$ and $\mathbf{F}(\Omega_j)$ (which must vanish asymptotically under frequency-specific effects) and that between $\hat{\mathbf{F}}(\Omega_j)$ and $\mathbf{F}$ (which must vanish asymptotically under no frequency-specific effects).

Accounting for the use of spectral regressions and closely related methods for the analysis of frequency-specific effects in economics and finance goes beyond the scope of this paper. We refer to the recent survey of Bandi and Tamoni (2022) for an up-to-date, comprehensive discussion of this vast strand of literature.

2.2.2. Simulation results

We generate $r = 2$ common factors $\mathbf{F}_t = A\mathbf{F}_{t-1} + \eta_t$ with $A = diag\,(0.4, 0.4)$, and idiosyncratic errors $e_{it} = 0.8\varepsilon_{it} + 0.2\epsilon_t$ where η_t, ε_{it} and ϵ_t are mutually independent *iid* $\mathrm{N}(0,1)$. So we have autocorrelated factors and weakly cross-sectional dependent errors. A $T \times N$ panel $\mathbf{X}$ is generated as described in equation (4) with $\Omega_2 = [-\theta, \theta]$, and $\Omega_1 = [-\pi, -\theta) \cup (\theta, \pi]$ considering three different scenarios.

DGP 1 : $\theta = \pi/2$, Λ_1, Λ_2 independently drawn from a uniform distribution in $[-1, 1]$.

DGP 2 : $\theta = \pi/4$, Λ_1 and Λ_2 independently drawn from a uniform distribution in $[-1, 1]$.

DGP 3 : $\theta = \pi/4$, $\Lambda_1 = \Lambda_2$ drawn from a uniform distribution in $[-1, 1]$.

We measure estimation accuracy by projecting estimated factors onto real ones and report trace-R^2 statistics

$$R^2\left(\hat{\mathbf{Y}}, \mathbf{Y}\right) = tr\left(\hat{\mathbf{Y}}' P_Y \hat{\mathbf{Y}}\right) / tr\left(\hat{\mathbf{Y}}'\hat{\mathbf{Y}}\right) \tag{10}$$

of such multivariate projections, where $tr(\cdot)$ stands for trace, $P_Y = \mathbf{Y}\left(\mathbf{Y}'\mathbf{Y}\right)^{-1}\mathbf{Y}'$, $\mathbf{Y}$ and $\hat{\mathbf{Y}}$ are either $\mathbf{F}$, $\mathbf{F}(\Omega_1)$, $\mathbf{F}(\Omega_2)$ and $\hat{\mathbf{F}}$, $\hat{\mathbf{F}}(\Omega_1)$, $\hat{\mathbf{F}}(\Omega_2)$, respectively. The results for each DGP and $T \times N = [25\ 50\ 100\ 200] \times [25\ 50\ 100\ 200]$ are obtained as averages across 500 replications.

The trace-R^2 statistics in Table 1 show that in presence of frequency-specific effects, that is under DGP 1 and DGP 2, the BSPC estimator yields mean-square consistent estimation of frequency-specific factors: as N and T grow $R^2\left(\hat{\mathbf{F}}(\Omega_1), \mathbf{F}(\Omega_1)\right)$ and $R^2\left(\hat{\mathbf{F}}(\Omega_2), \mathbf{F}(\Omega_2)\right)$ approach 1. On the contrary, under DGP 3, that is in absence of frequency-specific effects, the BSPC estimator estimates $\mathbf{F}_t$: $R^2\left(\hat{\mathbf{F}}(\Omega_1), \mathbf{F}\right)$ and $R^2\left(\hat{\mathbf{F}}(\Omega_2), \mathbf{F}\right)$ approach 1 while $R^2\left(\hat{\mathbf{F}}(\Omega_1), \mathbf{F}(\Omega_1)\right)$ and $R^2\left(\hat{\mathbf{F}}(\Omega_1), \mathbf{F}(\Omega_2)\right)$ do not. In fact, as discussed in Section 2.2.1, in this case the BSPC is a inefficient but consistent estimator of $\mathbf{F}$. Nonetheless, the loss of efficiency is very mild since the trace-R^2s of the two BSPC estimates, $R^2\left(\hat{\mathbf{F}}(\Omega_1), \mathbf{F}\right)$ and $R^2\left(\hat{\mathbf{F}}(\Omega_2), \mathbf{F}\right)$, are very close to those of the usual principal component estimator, $R^2\left(\hat{\mathbf{F}}, \mathbf{F}\right)$.

The first two DGPs violate the usual assumptions for the consistent principal component estimation of $\mathbf{F}$ and, indeed, $\hat{\mathbf{F}}_t$ does not seem to converge to $\mathbf{F}_t$ (this is particularly evident for DGP 2). DGP 3 is instead a usual factor model and the good performance of $\hat{\mathbf{F}}$ is in line with well-known results in the factor model literature.

The finite-sample properties of the BSPC estimator under the first two DGPs are somehow inferior to those of the PC estimator under DGP 3, but still reasonable for sufficiently large N and T. For example, in Figure 1 we repeat the same exercise for $r = 1$ and $N = T = 200$ and find that true and estimated factors are nearly undistinguishable. The solid lines are the spectra obtained with the unfeasible lag-window estimator that uses true factors, the dashed lines are instead obtained using the factors estimated via BSPCs. In the first two DGPs the estimated spectra of $\hat{F}_t(\Omega_1)$ and $\hat{F}_t(\Omega_2)$ are very close to those obtained using the unfeasible estimator which observes $F_t(\Omega_1)$ and $F_t(\Omega_2)$. Under DGP 3 the estimated spectra of $\hat{F}_t(\Omega_1)$ and $\hat{F}_t(\Omega_2)$ are undistinguishable because $\hat{F}_t(\Omega_1)$ and $\hat{F}_t(\Omega_2)$ are both estimates of F_t (the confidence bands for $\hat{F}_t(\Omega_1)$ and $\hat{F}_t(\Omega_2)$ are also remarkably similar).

2.3. Forecasting bond returns: supervised learning and band spectrum principal components

The use of principal component analysis in economics and finance is widespread because generally the space spanned by a high-dimensional process $\mathbf{X}_t = (x_{1t}, x_{2t}, \cdots, x_{Nt})$, such as a collection of macroeconomic variables, is well approximated by that spanned by a small number of principal components $\left(\hat{F}_{1t}, \hat{F}_{2t}, \cdots, \hat{F}_{rt}\right)$, with $r \ll N$. Indeed, principal components are widely used to estimate unobservable common factors (Bai and Ng, 2002; Forni et al., 2000; Stock and Watson, 2002a) and predict macroeconomic aggregates (see Stock and Watson, 2002b; Giannone et al., 2008; Forni et al., 2018, among many others).

Predicting a specific target, such as excess bond returns, is a different problem than fitting a collection of macroeconomic variables or aggregates. Even if the macroeconomy contains predictive information for bond returns, some common macroeconomic factors may represent macroeconomic fluctuations unrelated to bond returns. In this case, it becomes necessary to identify a subspace the predictive signal lives in which is spanned by a subset of common factors. For example, Ludvigson and Ng (2009) perform a model selection procedure for which 8 principal components and powers thereof are considered in the minimisation of a BIC criterion. Their selected specification is based on a linear combination of $\left(\hat{F}_{1t}, \hat{F}_{1t}^3, \hat{F}_{3t}, \hat{F}_{4t}, \hat{F}_{8t}\right)$.

The problem of estimating a predictive signal living in a common factor subspace has been widely considered in the statistical learning literature. Supervised statistical learning solves this problem in a simpler manner by embedding the individual predictive power of each covariate $x_{1t}, x_{2t}, \ldots, x_{Nt}$ into the extraction of the predictive signal via principal components. Using correlation as a measure predictive power, Bair et al. (2006) estimate predictors as principal components of a subset of covariates that correlate well with the predictive target.Another strand of this literature is based on the idea of sufficiency for estimating a minimal common factor subspace, which is referred to as the *central subspace* (Cook, 2007). This subspace is minimal because, despite dimension reduction, it contains all the information in the covariates for the predictive target.[12] These methods are based on the projection of each covariate x_{it} onto proxies for the central subspace, such as the observed past of the predictive target (Cook and Forzani, 2008; Huang et al., 2022, 2023) and/or other observed variables (Fan et al., 2017, 2021). Principal components are then applied to the projected (or fitted) values of the covariates.

In a linear predictive model the dimension of the central subspace is one.[13] For example, in the model $rx_{t+1} = \mu + \lambda'_{rx}\mathbf{F}_t + \varepsilon_{t+1}$ (where λ_{rx} may have one or more zero elements), the central subspace is $\text{Span}\left(\lambda'_{rx}\mathbf{F}_t\right)$ and $\lambda'_{rx}\mathbf{F}_t$ is a single sufficient index. If, also, $\mathbf{F}_t$ obeys a standard factor model (1), one principal component of projected data is sufficient. This is so because, if $\mathbf{z}_t$ is a vector of proxies for the central subspace, each projection $\hat{x}_{it}\left(\mathbf{z}\right) = \text{Proj}\left(x_{it}|\mathbf{z}_t\right)$ of x_{it} onto $\mathbf{z}_t$ has a common component which is proportional to $\lambda'_{rx}\mathbf{F}_t$. Therefore, the first principal component of $\hat{x}_{1t}\left(\mathbf{z}\right), \hat{x}_{2t}\left(\mathbf{z}\right), \cdots, \hat{x}_{Nt}\left(\mathbf{z}\right)$ estimates the sufficient single factor $\lambda'_{rx}\mathbf{F}_t$.

In this work we investigate whether the central subspace for excess bond returns is spanned by frequency-specific factors. Consider, for example, $rx_{t+1} = \mu + \lambda'_{rx}\mathbf{F}_t\left(\Omega\right) + \varepsilon_{t+1}$. In this scenario, the central subspace becomes $\text{Span}\left(\lambda'_{rx}\mathbf{F}_t\left(\Omega\right)\right)$ and the sufficient single factor $\lambda'_{rx}\mathbf{F}_t\left(\Omega\right)$

[12] That is, the conditional distribution of the target given the predictors $\mathbf{X}$ is the same as that given a lower-dimensional transformation of $\mathbf{X}$.

[13] The dimension of the central subspace is greater than one in the nonlinear case considered by Fan et al. (2017).

is estimated by a band spectrum principal component at the band Ω of $\hat{x}_{1t}(\mathbf{z})$, $\hat{x}_{2t}(\mathbf{z})$, $\cdots$, $\hat{x}_{Nt}(\mathbf{z})$. This example can be extended by allowing for frequency-specific factors in different bands, such as $rx_{t+1} = \mu + \lambda'_{rx,1}\mathbf{F}_t(\Omega_1) + \lambda'_{rx,2}\mathbf{F}_t(\Omega_2) + \varepsilon_{t+1}$. In this case, allowing for $\lambda'_{rx,1}\mathbf{F}_t(\Omega_1)$ to be proxied by $\mathbf{z}_t^{(1)}$ and $\lambda'_{rx,2}\mathbf{F}_t(\Omega_2)$ by $\mathbf{z}_t^{(2)}$, for the central subspace we need one band spectrum principal component in Ω_1 of projected data $\hat{x}_{1t}\left(\mathbf{z}^{(1)}\right)$, $\hat{x}_{2t}\left(\mathbf{z}^{(1)}\right)$, $\cdots$, $\hat{x}_{Nt}\left(\mathbf{z}^{(1)}\right)$, and one in Ω_2 of $\hat{x}_{1t}\left(\mathbf{z}^{(2)}\right)$, $\hat{x}_{2t}\left(\mathbf{z}^{(2)}\right)$, $\cdots$, $\hat{x}_{Nt}\left(\mathbf{z}^{(2)}\right)$.

In full generality, our predictors are obtained as follows. Letting $\mathbf{z}_t$ now be a vector of proxies for the central subspace at frequencies in Ω, we take the projected values $\hat{x}_{it}(\mathbf{z})$ where $\hat{x}_{it}(\mathbf{z})$ is an estimate of the component of x_{it} driven by the subset of Ω-specific factors that predicts excess bond returns. Letting $\hat{\mathbf{C}}_{\hat{x},0}(\Omega,\mathbf{z})$ be the band spectrum covariance matrix of the $T \times N$ panel of projected data $\hat{\mathbf{X}}(\mathbf{z}) \coloneqq \{\hat{x}_{it}(\mathbf{z})\,;\ i = 1,\ldots,N;\ t = 1,\ldots,T\}$, we consider band spectrum principal components of projected data

$$\begin{aligned} v^*_{\hat{x}}(\Omega,\mathbf{z}) &= \underset{v\in\mathbb{R}^N,\ v'v=1}{\arg\max}\ v'\hat{\mathbf{C}}_{\hat{x},0}(\Omega,\mathbf{z})\,v \\ \hat{F}_t(\Omega,\mathbf{z}) &= v^*_{\hat{x}}(\Omega,\mathbf{z})'\,\hat{\mathbf{X}}_t(\mathbf{z}) \end{aligned} \tag{11}$$

where $\hat{\mathbf{X}}_t(\mathbf{z}) = (\hat{x}_{1t}(\mathbf{z}), \hat{x}_{2t}(\mathbf{z}), \cdots, \hat{x}_{Nt}(\mathbf{z}))'$. As discussed in Section 2.2, for the band spectrum covariance we use the plugin estimator

$$\hat{\mathbf{C}}_{\hat{x},0}(\Omega,\mathbf{z}) = \int_{\omega\in\Omega} \hat{\mathbf{S}}_{\hat{x}}(\omega)\,d\omega \tag{12}$$

where $\hat{\mathbf{S}}_{\hat{x}}(\omega)$ is the estimated spectral density matrix of $\hat{\mathbf{X}}(\mathbf{z})$ obtained using a lag-window estimator as in equation (9).

Algorithm: Projected Band Spectrum Principal Components

1. For $i = 1, \cdots, N$, take the projections $\hat{x}_{it}(\mathbf{z}) = \mathrm{Proj}(x_{it}|\mathbf{z}_t)$.
2. Estimate the spectral density $\mathbf{S}_{\hat{x}}(\omega)$ of $\hat{\mathbf{X}}(\mathbf{z})$ using a lag-window estimator (9).
3. Estimate the band-spectrum covariance matrix $\mathbf{C}_{\hat{x},0}(\Omega,\mathbf{z})$ as in equation (12).
4. Obtain $\hat{F}_t(\Omega,\mathbf{z})$ as in equation (11).

In the empirical part of this work, we predict one month ahead excess bond returns using the predictors $\hat{F}_t(\Omega,\mathbf{z})$ for different choices of Ω and $\mathbf{z}$.

Allowing for frequency-specific factors is the main element of novelty with respect to other existing supervised learning methods based on a large number of predictors with a common factor structure, projections onto observed proxies and principal component estimators. The closest approach to ours is that of Huang et al. (2022) which is also based on a linear forecasting equation. Fan et al. (2017) allow the predictive target to be some unknown nonlinear function of its sufficient predictive indices (whose dimension exceeds one because of nonlinearity), while Fan et al. (2021) also consider robust estimation based on Huber loss minimization. In both works observed covariates are exploited using a projected principal component estimator (Fan et al., 2016). Huang et al. (2023) further extend Fan et al. (2017) in order to allow for weak factors.

3. Data

3.1. Excess bond returns

The (continuously compounded) yield of a n-year bond is

$$y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$$

where $p_t^{(n)} = \ln P_t^{(n)}$, and $P_t^{(n)}$ denotes the time t nominal price of a bond with n-years left to maturity. The excess return of a risky n-year bond is given by the difference between the log return from a n-year bond bought at time t and sold m months later, and the yield on a m-period risk-free rate at time t.

$$rx_{t+m}^{(n)} = p_{t+m}^{\left(n-\frac{m}{12}\right)} - p_t^{(n)} - \frac{m}{12} y_t^{\left(\frac{m}{12}\right)} = n y_t^{(n)} - \left(n - \frac{m}{12}\right) y_{t+m}^{\left(n-\frac{m}{12}\right)} - \frac{m}{12} y_t^{\left(\frac{m}{12}\right)} \tag{13}$$

where m is the holding period and $y_t^{\left(\frac{m}{12}\right)}$ is the annualized m-period risk-free rate.

Setting $m = 1$, we construct (monthly) nonoverlapping excess bond returns. In so doing, we follow recent works (such as Gargano et al., 2019; Wan et al., 2022; Borup et al., 2023) which avocate the use of nonoverlapping returns versus the commonly used monthly overlapping returns corresponding to an annual holding period ($m = 12$). Generally, there are a number of reasons for doing so. First, there are important short-lived dynamics in excess bond returns, such as Lehman Brothers' bankruptcy, which cannot be captured with annual holding periods. Second, overlapping returns present difficulties with the turning points of business cycles, which bear an intimate relationship with return predictability. Third, nonoverlapping returns are free from the inferencial problems with overlapping returns described by Bauer and Hamilton (2018). More specifically, in this work we are interested in characterising the predictability of bond returns related to macroeconomic cycles of different lengths: adopting overlapping returns would impair the analysis of cycles shorter than 1 year.

Yield data is taken from the zero-coupon Treasury yield curve dataset of Liu and Wu (2021) considering maturities up to 10 years. This is the same choice as in works conducting a similar out-of-sample predictive exercise, such as Bianchi et al. (2021), Fan et al. (2022). This dataset is obtained using a nonparametric kernel-smoothing method which compares favourably to the popular alternative dataset of Gürkaynak et al. (2007) as it takes into account Treasury bills and securities with less than 3 months to maturity and is found to contain smaller pricing errors.[14]

[14] The popular dataset of Fama and Bliss (1987) is instead unfit to our analysis since it starts from the 1-year maturity, hence it cannot be used to contruct nonoverlapping returns.

3.2. Real-time macroeconomic data

We obtain real-time macroeconomic data from the ALFRED database published by the Federal Reserve Bank of St. Louis. Apart from minor differences due to discontinued variables, our dataset is similar to that adopted by Ghysels et al. (2018) and Wan et al. (2022).[15] We observe $N = 54$ variables which can be broadly classified as "output and income", "labor market", "housing", "money and credit", "prices". Most of these variables are nonstationary and need being transformed to achieve stationarity. After these transformations, reported in Appendix B, the sample observations available span from August 1972 until December 2020. Some variables are available at earlier dates, however this is the largest sample available without missing values.

We observe a total of 465 vintages running from April 1982 to December 2020.[16] With no ragged-edge data, our first vintage dated April 1982 would be based on 117 data points from August 1972 onwards. However, these variables are available with a publication delay, typically one or two months. For example, our April 1982 vintage contains variables observed from August 1972 until to March 1982 and some from August 1972 until to February 1982. For each vintage we cope with this problem by discarding the first few observations of the variables with a shorter publication delay until a balanced panel is obtained. This leaves us, for example, with an April 1982 vintage of our full dataset collecting all variables with an actual sample size of 115 observations.

Finally, we remove outliers without looking into the future and standardise the data before the estimation of our predictors.[17]

As a preliminary investigation into our macroeconomic dataset, we use the full sample in our latest vintage dated December 2020 of dimension $(T, N) = (579, 54)$ to decompose the covariance matrix into its components in the frequency bands indicated below.

$\mathit{\Omega}_1 = [2\pi/12, \pi]$ corresponding to cycles of length up to 1 year;

$\mathit{\Omega}_2 = [2\pi/36, 2\pi/12]$ corresponding to cycles of length between 1 and 3 years;

$\mathit{\Omega}_3 = [2\pi/96, 2\pi/36]$ corresponding to cycles of length between 3 and 8 years;

$\mathit{\Omega}_4 = [0, 2\pi/96]$ corresponding to cycles of length of 8+ years,

where, to simplify the notation, the italic $\mathit{\Omega} = [\underline{\omega}, \overline{\omega}]$ denotes the band $\Omega = [-\overline{\omega}, -\underline{\omega}] \cup [\underline{\omega}, \overline{\omega}]$ with $0 \leq \underline{\omega} < \overline{\omega} \leq \pi$. In Figure 2, for each band we show the normalized band spectrum covariance matrix $\mathcal{C}_0(\mathit{\Omega}) = 0.5\,(\overline{\omega} - \underline{\omega})^{-1}\,\mathbf{C}_0(\mathit{\Omega})$ of our dataset with variables grouped by five broad

[15] More precisely, 8 out of 60 variables used in Ghysels et al. (2018) were discontinued in December 2015 and, thus, we exclude them. Our dataset includes the remaining 52 variables plus *CURRDD* and *DEDEPSL* also used by Wan et al. (2022).

[16] Infrequently, two vintages of a variable are released in a month. In such cases we take the last vintage of the month. If no vintage is published in a month we take the last vintage of the previous month.

[17] For a given vintage, we define as outliers observations with absolute value larger than 6 times the interquartile distance and replace them with the median, where both interquantile distance and median are calculated from the empirical density in that vintage.

categories.[18] All in all, stronger comovements are visible as lower frequencies are considered — this is particularly evident for housing variables. So, while Granger (1966) described a typical spectral shape for which the long cycles are relatively more important in the (univariate) spectra of economic variables, we find that the strength of comovements among economic variables has a similar pattern. Other interesting patterns emerge from this picture such as the nearly constant covariances over all bands of "money and credit variables", and the covariance within the "prices" category, which is somewhat stronger at the two extreme bands $\varOmega_1$ and $\varOmega_4$.

In order to shed more light on the eigenstructure of the data, we now analyze if factor estimates within spectral bands differ from their full spectra counterpart, as usual estimated via principal components. As shown in our simulation exercise of Section 2.2, in absence of frequency-specific effects factors estimated at different frequency bands are estimates of full-spectra factors. In Table 2 we report trace-R^2 statistics $R^2\left(\hat{\mathbf{F}}\left(\varOmega_i\right), \hat{\mathbf{F}}\right)$ as in equation (10) for $i = 1, \ldots, 4$, where $\hat{\mathbf{F}}\left(\varOmega_i\right)$ is a $T \times r$-dimensional matrix of band spectrum principal components and $\hat{\mathbf{F}}$ (full-spectra) principal components. These results show that principal component estimates vary across bands of frequencies and the usual full-spectrum estimator is dominated by high-frequency comovements. The trace-R^2 between our lowest frequency BSPC and PC estimates takes values which are similar to those we found in the simulation exercise in presence of frequency-specific factors.

While these results cast some doubts on the lack of frequency-specific effects, this is far from being clear-cut evidence. Most importantly, this preliminary analysis of our macroeconomic dataset is not insightful on the predictive power of (BS)PC for excess bond returns.

4. Excess bond returns forecasts

We forecast nonoverlapping excess bond returns (13) one month ahead via usual predictive regressions of the type

$$\hat{rx}_{t+1}^{(n)} = \hat{\alpha} + \hat{\beta}\hat{F}_t\left(\Omega, \mathbf{z}\right) \tag{14}$$

where $\hat{F}_t\left(\Omega, \mathbf{z}\right)$ is a supervised band spectrum principal component obtained as in equation (11) for some choice of Ω and $\mathbf{z}$ to be discussed below.[19]

Our predictions are obtained estimating the forecasting equation (14) over an expanding window, that is, at time t we use all past data available in real time t which, as explained in Section 3.2, generally means using observations up to month $t-2$ or $t-1$.

Our first prediction is made at the time of our first vintage of April 1982 to predict the excess bond returns in May 1982 and so on until the last prediction made using the November

[18] The normalization by the size of the band gives comparable covariances $\mathcal{C}_0\left(\varOmega\right)$s across bands of different sizes. Their non-normalized counterparts $\mathbf{C}_0\left(\varOmega\right)$s are, instead, the components of $\mathbf{C}_0$ due to the fluctuations in different bands. For example, with our chosen bands we have $\mathbf{C}_0 = \sum_{i=1}^{4}\mathbf{C}_0\left(\varOmega_i\right) = \sum_{i=1}^{4} 2\left(\overline{\omega}_i - \underline{\omega}_i\right)\mathcal{C}_0\left(\varOmega_i\right)$.

[19] The estimation of the spectral density matrix in equation (9) is defined with a bandwidth equal to the smallest integer near to the square root of the sample size. As explained in Section 3.2, publication delays dictate the actual sample size available for our expanding estimation in real time t. Hence, our bandwidth becomes $\lfloor\sqrt{\mathcal{T}_t}\rfloor$ where $\mathcal{T}_t$ is the actual sample size in t.

2020 vintage to predict the excess bond returns in December 2020. Denoting T_0 the time corresponding to April 1982 and T that corresponding to December 2020, our out-of-sample forecasts are made in real time $t = T_0, T_0+1, \ldots, T-1$.

The methodology described in Section 2, is based on two key choices: a band of frequencies Ω for our frequency-specific factors, and a vector of proxies $\mathbf{z}$ for the predictive signal in the common macroeconomic cycles corresponding to the frequencies in Ω. Similarly in spirit to previous works on frequency-, horizon- or scale-specific effects, in order to dissect the predictability of excess bond returns we explore different choices of Ω. For example, Bandi et al. (2019) study scale-specific predictability in predictive regressions under temporal aggregation over different horizons. In order to observe whether the predictive power of common macroeconomic cycles varies across frequency bands, we consider the bands Ω_1, Ω_2, Ω_3, Ω_4, as defined in Section 3.2.

For each band Ω_i, $i = 1, \ldots, 4$, we consider two alternative vectors of proxies $\mathbf{z}_t$, both including the average excess bond return across maturities $\bar{rx}_{t+1} = \frac{1}{9}\sum_{n=2}^{10} rx_{t+1}^{(n)}$.

$\mathbf{z}_t^{Infl} = (infl_t,\ \bar{rx}_{t+1})'$, where $infl_t = (1-L)^2\, CPI_t$ and CPI_t is the "Consumer Price Index for All Urban Consumers: All Items" taken from the ALFRED dataset.[20]

$\mathbf{z}_t^{Tms} = (tms_t,\ \bar{rx}_{t+1})'$, where tms_t is the term spread which we take from the dataset of Welch and Goyal (2008).

As discussed in Section 2.3, the supervised learning literature suggests the target variable to be predicted as a natural proxy for the central subspace. Since, following Cochrane and Piazzesi (2005), the same factors are used to predict excess bond returns across all maturities, in both choices above we consider the "average target" $\bar{rx}_{t+1}$ rather than each target $rx_{t+1}^{(n)}$. Of course, since $\bar{rx}_{t+1}$ leads the predictors x_{it}, these choices mean that at time t the projections $\hat{x}_{it}(\mathbf{z}) = \text{Proj}(x_{it}|\mathbf{z}_t)$ can be estimated up to time $t-1$. Inflation and term spread are well-known predictors of excess returns since at least Fama (1981) and Fama and French (1989).

In order to reach a conclusion regarding the existence of frequency-specific effects, we also make predictions based on full-spectrum principal components of projected data corresponding to $\Omega_0 = [0,\ \pi]$ for which cycles of all lengths are aggregated.

For $i = 0, 1, \ldots, 4$, the predictions obtained using the predictor $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Infl}\right)$ in the forecasting equation (14) are denoted as $Infl(\Omega_i)$, while $Tms(\Omega_i)$ stands for the predictions using $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Tms}\right)$.

4.1. Statistical accuracy

We compare our forecasts against the standard benchmark suggested by the expectations hypothesis, the historical mean $\hat{rx}_{t+1,EH}^{(n)}$. Following Campbell and Thompson (2008), we use the

[20]Since CPI is part of our macroeconomic dataset, using it as a proxy means removing it from the panel $\mathbf{X}$ before the estimation of factors. Clearly, not doing so would yield a panel of projected data $\hat{\mathbf{X}}$ with a singular convariance matrix.

out-of-sample R^2 measure

$$R^2_{OS} = 1 - \frac{\sum_{t=T_0+1}^{T} \left(\hat{rx}_t^{(n)} - rx_t^{(n)} \right)^2}{\sum_{t=T_0+1}^{T} \left(\hat{rx}_{t,EH}^{(n)} - rx_t^{(n)} \right)^2} \tag{15}$$

that is, a relative reduction in mean square error, which in all tables is reported in percentages. Following the standard practice in this literature, we evaluate the statistical significance of these mean square error improvements using the test of Clark and West (2006).

The R^2_{OS} values in Table 3 support the existence of frequency-specific predictors as the forecasts corresponding to the bands Ω_2, Ω_3 and Ω_4 are considerably more accurate than those corresponding to Ω_1 and the full spectrum Ω_0. This suggests that high-frequency macroeconomic fluctuations are noisy and add measurement error to the predictors.

Starting from the forecasts obtained using the vector of proxies $\mathbf{z}_t^{Infl}$, while the R^2_{OS}s of $Infl\,(\Omega_0)$ and $Infl\,(\Omega_1)$ are either negative or insignificant at all maturities, $Infl\,(\Omega_2)$, $Infl\,(\Omega_3)$, $Infl\,(\Omega_4)$ provide large R^2_{OS}s at all maturities which are 1% significant at maturities 2 to 6 and 5% significant at maturities 7 to 10. Also, the R^2_{OS}s of $Infl\,(\Omega_2)$, $Infl\,(\Omega_3)$, $Infl\,(\Omega_4)$ are larger at shorter maturities. Overall, $Infl\,(\Omega_4)$ is slightly more accurate than $Infl\,(\Omega_2)$ and $Infl\,(\Omega_3)$ at each maturity.

Much statistical significance is found across all bands when $\mathbf{z}_t^{Tms}$ is used as proxy. However, 1% significance at all maturities is only found for $Tms\,(\Omega_2)$, $Tms\,(\Omega_3)$ and $Tms\,(\Omega_4)$. Furthermore, $Tms\,(\Omega_0)$ and $Tms\,(\Omega_1)$ are associated with smaller R^2_{OS}s at all maturities and considerably so for maturities of at least 3 years. $Tms\,(\Omega_2)$, $Tms\,(\Omega_3)$ and $Tms\,(\Omega_4)$ provide their largest R^2_{OS}s at maturities longer than 3 years.

These results provide evidence of frequency-specific predictability which vanishes completely or becomes weaker when full-spectrum predictors are used. In Appendix C (Table C.1) we show that even adopting the supervised learning approaches of Huang et al. (2023) and Fan et al. (2017) (which, as discussed in Section 2.3, are related to ours) full-spectrum predictors perform poorly. Regarding the real-time forecasting performance of unsupervised principal component predictors, we refer to Wan et al. (2022) who find no evidence of predictability for nonoverlapping returns. Finally, in unreported results (available upon request), we find that unsupervised frequency-specific factors are also not accurate. This confirms the results of Ludvigson and Ng (2009) who document the existence of common macroeconomic variation unrelated to bond return predictability.

4.2. Economic value of the forecasts

Thus far we found statistical evidence of bond return predictablility using frequency-specific factors. However, as pointed out by works such as Thornton and Valente (2012) and Sarno et al. (2016), statistically accurate forecasts do not necessarily generate economic value for inverstors trading on Treasury bonds. Therefore, we now examine whether our forecasts translate into economic gains for investors with mean-variance preferences or a power utility function. In both cases, we consider the asset allocation decisions of an investor who selects weights $w_t^{(n)}$ on a risky bond with n years to maturity versus the one-month T-bill, that is a risk-free yield $y_t^{(1/12)}$.

A mean-variance investor maximizes the utility function

$$U\left(w_t^{(n)}, rx_{t+1}^{(n)}\right) = E_t\left(R_{p,t+1}^{(n)}\right) - \frac{\gamma}{2}\mathrm{Var}_t\left(R_{p,t+1}^{(n)}\right) \tag{16}$$

where γ is the relative risk aversion and $R_{p,t+1}^{(n)} = y_t^{(1/12)} + w_t^{(n)} rx_{t+1}^{(n)}$ the portfolio return at time $t+1$ given the generic allocation $w_t^{(n)}$. The solution of the above optimisation problem is

$$\dot{w}_t^{(n)} = \gamma^{-1}\frac{\hat{rx}_{t+1}^{(n)}}{\left(\hat{\sigma}_{t+1|t}^{(n)}\right)^2}$$

where $\hat{rx}_{t+1}^{(n)}$ is some excess return forecast on n-year bond, and $\left(\hat{\sigma}_{t+1|t}^{(n)}\right)^2$ is the conditional variance estimated using a rolling window estimator over the past five years of observations as in Campbell and Thompson (2008).

A power utility investor instead maximizes the utility function

$$U\left(w_t^{(n)}, rx_{t+1}^{(n)}\right) = \frac{1}{1-\gamma}\left(\left(1 - w_t^{(n)}\right)\exp\left(y_t^{(1/12)}\right) + w_t^{(n)}\exp\left(y_t^{(1/12)} + rx_{t+1}^{(n)}\right)\right)^{1-\gamma} \tag{17}$$

In this case, the optimal weights we use are those obtained under the log-normal approximation of Campbell and Viceira (1999)

$$\dot{w}_t^{(n)} = \frac{1}{\gamma\left(\hat{\sigma}_{t+1|t}^{(n)}\right)^2}\left[\hat{rx}_{t+1}^{(n)} + \left(\hat{\sigma}_{t+1|t}^{(n)}\right)^2/2\right]$$

Under both preferences, we follow Campbell and Thompson (2008) who windorise the weights by imposing the restriction $0 \leq w_t^{(n)} \leq 1.5$ to prevent the investor from taking extreme positions such as leveraging above 150% and shorting positions.

The optimal portfolio weights $\dot{w}_t$ given some predictions $\hat{rx}_{t+1}^{(n)}$ are used at every time t to compute the investor's realized utilities $\dot{U}_{t+1}$. Similarly, the benchmark realized utilities $\dot{U}_{t+1,EH}$ are obtained using optimal weights given the expectations hypothesis forecasts $\hat{rx}_{t+1,EH}^{(n)}$. The certainty equivalent return (CER) gains of a given predictive model with respect to the benchmark are obtained as the difference between its average realized utility over time and the average benchmark realized utility. So, positive CER gains indicate that the predictive model considered produces economic value in excess of that of the expectations hypothesis model. We report CER gains in annualized percentage terms. Finally, to test whether these gains are statistically greater than zero, we use the test of Diebold and Mariano (1995). Specifically, we estimate the regression

$$\dot{U}_{t+1}^{(n)} - \dot{U}_{t+1,EH}^{(n)} = \delta^{(n)} + \epsilon_{t+1}^{(n)}$$

and test if $\delta^{(n)}$ equals zero. To examine the effect of risk aversion γ, we repeat the above analysis considering the values 3, 5 and 8.

Table 4 shows the CER gains for investors with mean-variance utility. The most important result here is the evidence of significant CER gains which thus far, to the best of our knowledge, has not been found with nonoverlapping returns using data available in real-time. However, no single predictor provides significant CER gains at all maturties and across all risk aversion coefficients. For example, no prediction is significant at maturties 9 and 10 when $\gamma = 8$.

Similarly to the R^2_{OS}s in Section 4.1, when $\mathbf{z}_t^{Infl}$ is used we find evidence of frequency-specific effects with results varying much across our spectral bands. Regardless the risk aversion coefficient, all CER gains of $Infl\,(\Omega_0)$ and $Infl\,(\Omega_1)$ are either negative or insignificant. The CER gains of $Infl\,(\Omega_2)$, $Infl\,(\Omega_3)$ and $Infl\,(\Omega_4)$ are instead significant across all maturities for $\gamma = 3$, until maturity 8 for $\gamma = 5$, and until maturity 6 for $\gamma = 8$. At least for $\gamma = 5, 8$, $Infl\,(\Omega_4)$ is slightly better than $Infl\,(\Omega_2)$ and $Infl\,(\Omega_3)$.[21]

Some interesting patterns across our spectral bands emerge when $\mathbf{z}_t^{Tms}$ is used. For all risk aversion coefficients, $Tms\,(\Omega_1)$ gives significant CER gains at maturities 2 to 6 and insignificant gains at maturities 8 to 10. $Tms\,(\Omega_2)$ has the largest (significant) CER gains at maturities 7 to 10 for risk aversion $\gamma = 3, 5$, and at maturities 5 to 8 for $\gamma = 8$. Despite being outperformed by $Tms\,(\Omega_2)$, $Tms\,(\Omega_3)$, $Tms\,(\Omega_4)$ in statistical terms, $Tms\,(\Omega_0)$ generates some significant CER gains, especially at the two shortest maturities, 2 and 3.

At least qualitatively, the results are very similar for power utility investors; the corresponding CER gains are reported in Appendix C (Table C.2).

4.3. Two predictors and the spanning hypothesis

In the previous sections we found considerable differences between forecasts obtained across different bands of frequencies both in statistical and economic terms. This is true for both families of predictors $\hat{F}_t\left(\Omega, \mathbf{z}^{Infl}\right)$ and $\hat{F}_t\left(\Omega, \mathbf{z}^{Tms}\right)$ as Ω varies between the bands Ω_1 to Ω_4. Nonetheless, $Tms\,(\Omega)$ predictions are better when shorter common macroeconomic cycles are considered and are relatively more accurate for bonds with longer maturities, while the opposite applies to $Infl\,(\Omega)$. So, we are tempted to conjecture that $\mathbf{z}^{Infl}$ and $\mathbf{z}^{Tms}$ proxy for different predictable components of excess bond returns.

Searching for more convincing evidence, we now extend our out-of-sample predictive exercise by considering the following additional predictive models based on two predictors

$$\hat{rx}_{t+1}^{(n)} = \hat{\alpha} + \hat{\beta}_1 \hat{F}_t\left(\Omega_i, \mathbf{z}^{Infl}\right) + \hat{\beta}_2 \hat{F}_t\left(\Omega_j, \mathbf{z}^{Tms}\right) \tag{18}$$

for any possible pair of predictors for $i, j = 0, 1, \ldots, 4$. For each pair we make forecasts and in Figure 3 we report averages across maturities of R^2_{OS} values and CER gains under mean-

[21] Among $Infl\,(\Omega_2)$, $Infl\,(\Omega_3)$, $Infl\,(\Omega_4)$, the latter is the only one to exceed the others at some maturities by at least 0.01 and at the same or higher levels of significance.

variance preferences. These results show that the most accurate individual predictors combine well together. In line with the above results on $Infl\,(\mathit{\Omega}_4)$ and $Tms\,(\mathit{\Omega}_2)$, the predictions based on both predictors $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ and $\hat{F}_t\left(\mathit{\Omega}_2, \mathbf{z}^{Tms}\right)$ yield the largest average R^2_{OS} and average CER gains for all risk aversion coefficients. Also, similarly to the evidence on individual predictors, full-spectrum predictions — corresponding to the pair $\hat{F}_t\left(\mathit{\Omega}_0, \mathbf{z}^{Infl}\right)$, $\hat{F}_t\left(\mathit{\Omega}_0, \mathbf{z}^{Tms}\right)$ — or those based on the shortest macroeconomic cycles are less accurate.[22] Again, R^2_{OS}s and CER gains vary much across the bands of frequencies considered. $\hat{F}_t\left(\mathit{\Omega}, \mathbf{z}^{Infl}\right)$ gives much better results at $\mathit{\Omega}_2$, $\mathit{\Omega}_3$, $\mathit{\Omega}_4$, that is when higher-frequency fluctuations are excluded. $\hat{F}_t\left(\mathit{\Omega}, \mathbf{z}^{Tms}\right)$ is more accurate at $\mathit{\Omega}_2$, especially in terms of CER gains.

In Table 5, for each maturity we report all results — R^2_{OS} values and CER gains — based on the forecasts jointly produced by our most accurate predictors $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ and $\hat{F}_t\left(\mathit{\Omega}_2, \mathbf{z}^{Tms}\right)$ using the forecasting equation (18) for $i = 4$ and $j = 2$. We label such forecasts as *Both*. In Panel A of Table 5 we see that *Both* gives R^2_{OS} values which are considerably larger than $Infl\,(\mathit{\Omega}_4)$ and $Tms\,(\mathit{\Omega}_2)$ (or any other prediction obtained with a single predictor) and 1% significant at all maturities. The evidence in Panel B of Table 5 is even stronger since, unlike any forecast based on individual predictors, *Both* gives significant CER gains at all maturities and for any value of risk aversion. This strengthens our novel result of significant CER gains using nonoverlapping returns and data available in real time: *Both* forecasts are our best both in economic and statistical terms. We must conclude that $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ and $\hat{F}_t\left(\mathit{\Omega}_2, \mathbf{z}^{Tms}\right)$ are two powerful predictors for different predictive components of excess bond returns.

The results discussed thus far do not help understanding whether excess bond return predictability comes uniquely from the information contained in the current yield curve as the *spanninng hypothesis* postulates. While, by construction, $\hat{F}_t\left(\mathit{\Omega}_2, \mathbf{z}^{Tms}\right)$ is a spanned factor, our inflation factor $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ could contain information which is unspanned by the cross-section of yields. In order to determine if that is the case, we need to obtain an unspanned version of the inflation factor. We do so by controlling for yields variation in the estimation of the inflation factor. In our algorithm (Section 2.3) we now replace $\hat{\mathbf{X}}_t\left(\mathbf{z}\right)$ with $\tilde{\mathbf{X}}_t\left(\mathbf{z}\right) = \left\{\tilde{x}_{it}\left(\mathbf{z}\right), i = 1, \ldots, N\right\}$ where $\tilde{x}_{it}\left(\mathbf{z}\right) = \delta'_{i,z}\mathbf{z}_t$ is the unspanned component of x_{it} using some controls for yields $\mathbf{c}_t$ in the projection $\mathrm{Proj}\left(x_{it}|\mathbf{z}_t, \mathbf{c}_t\right) = \delta'_{i,z}\mathbf{z}_t + \delta'_{i,c}\mathbf{c}_t$.[23] We repeat this exercise with four different controls for yields: the Cochrane-Piazzesi factor ($\mathbf{c}^{(1)}_t$), 3 and 5 principal components of yields ($\mathbf{c}^{(2)}_t$ and $\mathbf{c}^{(3)}_t$), and our term spread factor $\hat{F}_t\left(\mathit{\Omega}_2, \mathbf{z}^{Tms}\right)$ (now $\mathbf{c}^{(4)}_t$). The R^2_{OS} values in Table 6 show that these four unspanned versions of our original inflation factor $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ generate predictions which are as accurate as those generated by the original factor (already seen in Table 3 and repeated in the last line of Table 6 for readability). Therefore, all the predictive content of the inflation factor $\hat{F}_t\left(\mathit{\Omega}_4, \mathbf{z}^{Infl}\right)$ must reside in its unspanned component. These results imply

[22] Despite positive averages across maturities, in unreported results (available upon request), we found that the full-spectrum pair $\hat{F}\left(\mathit{\Omega}_0, \mathbf{z}^{Infl}\right), \hat{F}\left(\mathit{\Omega}_0, \mathbf{z}^{Tms}\right)$ and the pair at our highest-frequency band $\hat{F}\left(\mathit{\Omega}_1, \mathbf{z}^{Infl}\right), \hat{F}\left(\mathit{\Omega}_1, \mathbf{z}^{Tms}\right)$ provide little evidence of significant CER gains across maturities and risk aversion coefficients.

[23] An alternative procedure would be to first project the x_{it}'s onto $\mathbf{c}_t$ and then estimate the inflation factor using the residuals of such projection. The results are very similar to those in Table 6.

that affine term structure models should include macroeconomic unspanned information which, similarly to Ludvigson and Ng (2009), Chernov and Mueller (2012), Joslin et al. (2014), Coroneo et al. (2016), relates to the inflation.[24]

5. Links to the real economy

Motivated by the intuition that investors demand compensation for the risk of recessions, notable rational expectations models, such as Campbell and Cochrane (1999) and Wachter (2006), feature countercyclical risk aversion. Furthermore, countercyclical risk premia have been widely documented by prior empirical works dismissing the expectations hypothesis such as Fama and Bliss (1987) and Campbell and Shiller (1991). Having established evidence of predictability, we now investigate whether our out-of-sample expected returns are consistent with the theory of countercyclical risk premia.

We start by measuring whether our expected returns correlate with monthly measures of real economic activity growth. As seen in Table 7, expected returns generated by our two most accurate predictors $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$, $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$ either individually ($Infl\,(\Omega_4)$ and $Tms\,(\Omega_2)$) or jointly ($Both$) are clearly countercyclical. All these forecasts are negatively correlated with the Michigan consumer sentiment index (MCSI) and significantly so at 1% for all maturities. Evidence of counterciclicality is also found by looking at the year-on-year industrial production growth (IP y-o-y growth) and the Chicago Fed National Activity sub-index on consumption and housing (CFNAI C&H). The only exception is the correlation between $Tms\,(\Omega_2)$ and IP y-o-y growth which is still negative but insignificant. The absolute value of all correlations increases across maturities. Similar evidence is obtained by adjusting for risk: in Table C.3 (Appendix C) we report larger Sharpe ratios during recessions than expansions for all maturities and predictions $Infl\,(\Omega_4)$, $Tms\,(\Omega_2)$, $Both$.

Following Ludvigson and Ng (2009), we extend this analysis by considering the term premium, defined as the gap between an n-year yield $y_t^{(n)}$ and its expectation component $n^{-1}E_t\left(y_t^{(1)} + y_{t+1}^{(1)} + \cdots + y_{t+n-1}^{(1)}\right)$, which can be estimated as

$$tp_t^{(n)} = \frac{1}{n}\left(\hat{rx}_{t+12}^{(n)} + \hat{rx}_{t+24}^{(n-1)} + \cdots + \hat{rx}_{t+12(n-1)}^{(2)}\right) \tag{19}$$

where the hats stand for predictions at time t. While the expectations hypothesis implies a constant term premium, rational expectation models with time-varying risk aversion instead predict a countercyclical term premium. Adopting the standard VAR procedure for multi-step ahead forecasts introduced by Ludvigson and Ng (2009) and followed by many works closely related to ours (Ghysels et al., 2018; Huang et al., 2023), we measure the cyclical properties of the term premium implied by our $Both$ forecasts.[25] In Figure 4 we show the term premium

[24] Note, however, some of these works also propose other macroeconomic unspanned factors related to the real activity.

[25] Following Ludvigson and Ng (2009) h-year-ahead predictions are obtained using a monthly vector autoregres-

estimated using all maturities considered so far, that is $n = 10$, excluding or including our predictors $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$, $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$ (top to bottom) against IP y-o-y growth. Both estimated term premia are countercyclical but the countercyclicality obtained including our predictors is almost twice as large — i.e. the correlation between the estimated term premium and industrial production growth is -0.29 versus -0.17 — similarly to the results of Bianchi et al. (2021). All correlation coefficients are 1% significant.

Adopting another popular way in the literature to study the cyclical pattern of predictability, we now split our sample into periods of recessions and expansions using the NBER recession indicator and observe how our forecasts perform in these two subsamples.[26] Mirroring the analysis in Section 4, we evaluate our forecasts in both statistical and economic terms. In Table 8 we decompose the R^2_{OS} of the models using $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$, $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$ either individually ($Infl\,(\Omega_4)$ and $Tms\,(\Omega_2)$) or jointly ($Both$). Most predictability is found in recessions. However, unlike a number of works concluding that return predictability is concentrated in recessions and absent during expansions — among many others see Rapach and Zhou (2013), Henkel et al. (2011), Dangl and Halling (2012) for equity returns, and Sarno et al. (2016); Gargano et al. (2019) for bond returns —, we find some evidence of predictability even in expansions. Nonetheless, this result is not unprecedented in the literature. In fact, the same finding is reported by Bianchi et al. (2021) who also use machine learning techniques, albeit different from ours. Even more interestingly, Table 8 clarifies that predictability in expansions is entirely caught by one predictor, $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$, which highlights two remarkable similarities with Andreasen et al. (2021) regarding the performance of term structure predictors across the business cycle. First, they are powerful during expansions. Second, the forecasts they generate are less accurate during recessions for longer maturities. The only forecasts in Table 8 obtained without including $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$, i.e. $Infl\,(\Omega_4)$, generate negative R^2_{OS} in expansions at most maturities, while $Tms\,(\Omega_2)$ and $Both$ give 1%-significant improvements over the benchamark at all maturities.

Table 8 also shows that accuracy in recessions is way higher than during expansions and, conversely to expansions, it is due to our other predictor, $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$. In line with the above results, R^2_{OS}s are much larger during recessions. The same conclusions are suggested by Table 9 where we report the CER gains in expansions and recessions: larger economic value is observed during recessions and is mostly generated by $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$, whereas smaller but significant economic value is generated by $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$ during expansions.

6. Conclusions

Inspired by mounting evidence of frequency-specific risk, we develop a new method to study the predictability of bond returns using frequency-specific latent macroeconomic factors estimated in real time. Disentangling macroeconomic cycles of different lengths proves to be important.

sive model with 12 lags that includes as variables the excess bond returns and a set of predictors.

[26] The recession indicator “USREC”, is taken from the FRED database.

While full spectrum predictors are inaccurate or suboptimal, two powerful predictors are found by exploring specific bands of frequencies: an inflation factor related to common macroeconomic cycles of at least 8 years, and a term spread factor related to macroeconomic cycles of 1 to 3 years. Our inflation factor contains information unspanned by current yields and forward rates, is relatively more powerful for shorter maturities and generates much predictability during recessions. Our term spread factor is instead relatively more powerful for longer maturities and generates significant predictability during expansions.

Apart from predictive accuracy, our forecasts generate significant economic value for investors and are in line with countercyclical risk aversion and unspanned macroeconomic information in the yield curve. Other works found evidence of predictability, countercyclical risk aversion and unspanned macroeconomic factors, however ours is the first one doing so obtaining forecasts with economic value based on real-time data and nonoverlapping returns. Therefore, we confirm the big picture of Ludvigson and Ng (2009) while taking into account the reasonable concerns on the use of revised macroeconomic data (Ghysels et al., 2018) and the drawbacks of overlapping returns, and overcoming the difficulty in translating forecasting accuracy into economics gains (Thornton and Valente, 2012; Sarno et al., 2016) which is found to affect even deep learning methods (Fan et al., 2022).

Other interesting results emerge thanks to the frequency-specific nature of our analysis. Our inflation factor is consistent with the long-run risk model of Bansal and Shaliastovich (2013). Our term spread factor has cyclical properties similar to those found by Fama and French (1989) and its performance is very reminiscent of the results of Andreasen et al. (2021) on the predictive power of yield curve predictors during expansions. Finally, our two predictors are in line with the yield decomposition of Cieslak and Povala (2015) based on a slow-moving average of inflation and a cycle factor obtained as the component of yields orthogonal to trend inflation whose predictive power, similarly to that of our term spread factor, increases across maturities.

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Tables

Table 1: Simulation results.

	DGP 1				DGP 2				DGP 3			
	$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\left(\Omega_1\right)\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\left(\Omega_1\right)\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\left(\Omega_1\right)\right)$			
	$N=25$	50	100	200	$N=25$	50	100	200	$N=25$	50	100	200
$T=25$	0.543	0.677	0.751	0.818	0.728	0.826	0.878	0.918	0.531	0.552	0.566	0.568
50	0.583	0.733	0.826	0.902	0.774	0.870	0.929	0.962	0.491	0.506	0.513	0.523
100	0.625	0.762	0.863	0.927	0.797	0.886	0.940	0.969	0.485	0.507	0.521	0.520
200	0.643	0.773	0.875	0.932	0.813	0.893	0.946	0.972	0.491	0.507	0.517	0.520
	$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\left(\Omega_2\right)\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\left(\Omega_2\right)\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\left(\Omega_2\right)\right)$			
	$N=25$	50	100	200	$N=25$	50	100	200	$N=25$	50	100	200
$T=25$	0.781	0.836	0.865	0.884	0.607	0.682	0.715	0.761	0.426	0.446	0.449	0.459
50	0.837	0.888	0.919	0.929	0.720	0.804	0.851	0.883	0.454	0.473	0.483	0.485
100	0.861	0.910	0.940	0.956	0.763	0.845	0.895	0.928	0.454	0.465	0.467	0.479
200	0.874	0.926	0.957	0.969	0.782	0.866	0.922	0.947	0.445	0.462	0.468	0.475
	$R^2\left(\hat{\mathbf{F}},\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}},\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}},\mathbf{F}\right)$			
	$N=25$	50	100	200	$N=25$	50	100	200	$N=25$	50	100	200
$T=25$	0.586	0.619	0.646	0.654	0.523	0.548	0.570	0.581	0.853	0.887	0.907	0.918
50	0.620	0.654	0.685	0.693	0.494	0.529	0.549	0.553	0.891	0.925	0.943	0.952
100	0.623	0.674	0.693	0.703	0.479	0.515	0.532	0.538	0.913	0.944	0.961	0.970
200	0.644	0.682	0.713	0.717	0.481	0.498	0.528	0.530	0.923	0.955	0.972	0.980
	$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_1\right),\mathbf{F}\right)$			
	$N=25$	50	100	200	$N=25$	50	100	200	$N=25$	50	100	200
$T=25$	0.375	0.380	0.361	0.349	0.496	0.528	0.533	0.538	0.844	0.881	0.903	0.914
50	0.340	0.315	0.303	0.289	0.459	0.480	0.489	0.498	0.886	0.922	0.941	0.950
100	0.317	0.303	0.285	0.282	0.456	0.481	0.499	0.506	0.911	0.942	0.960	0.969
200	0.314	0.286	0.284	0.273	0.461	0.489	0.506	0.513	0.921	0.955	0.971	0.980
	$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\right)$				$R^2\left(\hat{\mathbf{F}}\left(\Omega_2\right),\mathbf{F}\right)$			
	$N=25$	50	100	200	$N=25$	50	100	200	$N=25$	50	100	200
$T=25$	0.606	0.643	0.652	0.669	0.473	0.494	0.506	0.515	0.847	0.880	0.901	0.912
50	0.642	0.675	0.692	0.703	0.461	0.480	0.491	0.501	0.891	0.924	0.942	0.950
100	0.653	0.680	0.700	0.712	0.437	0.459	0.473	0.479	0.913	0.944	0.961	0.970
200	0.664	0.691	0.712	0.721	0.435	0.460	0.471	0.480	0.923	0.956	0.972	0.980

Notes: The table reports trace-R^2 statistics (10). Data generating processes and all details of the simulation exercise are described in Section 2.2.2. T,N denote the dimension of the panel considered for each DGP.

Table 2: MACRO DATA: BSPC ESTIMATES.

	r				
	1	2	3	5	8
$R^2\left(\hat{\mathbf{F}}(\Omega_1), \hat{\mathbf{F}}\right)$	0.997	0.910	0.878	0.905	0.962
$R^2\left(\hat{\mathbf{F}}(\Omega_2), \hat{\mathbf{F}}\right)$	0.953	0.941	0.904	0.935	0.904
$R^2\left(\hat{\mathbf{F}}(\Omega_3), \hat{\mathbf{F}}\right)$	0.748	0.738	0.791	0.873	0.866
$R^2\left(\hat{\mathbf{F}}(\Omega_4), \hat{\mathbf{F}}\right)$	0.617	0.713	0.773	0.849	0.850

Notes: Trace-R^2 statistics (10) for common factors estimated from the of the ALFRED dataset (December 2020 vintage). $\hat{\mathbf{F}}(\Omega_i)$ are band spectrum principal components where the bands Ω_i for $i = 1, \ldots, 4$ are defined in Section 3.2; $\hat{\mathbf{F}}$ are principal components.

Table 3: OUT OF SAMPLE R^2 ACROSS FREQUENCY BANDS.

	Maturities								
	2	3	4	5	6	7	8	9	10
$Infl\,(\Omega_1)$	0.593	0.279	0.136	−0.023	−0.150	−0.047	0.027	0.027	−0.023
$Infl\,(\Omega_2)$	4.385***	3.723***	2.418***	1.948***	1.993***	1.871**	1.885**	1.653**	1.573**
$Infl\,(\Omega_3)$	4.481***	3.904***	2.606***	2.115***	2.094***	1.936**	1.936**	1.687**	1.567**
$Infl\,(\Omega_4)$	4.578***	3.990***	2.679***	2.170***	2.142***	1.987**	1.982**	1.725**	1.597**
$Infl\,(\Omega_0)$	0.568	0.200	0.074	−0.106	−0.289	−0.143	−0.059	−0.049	−0.041
$Tms\,(\Omega_1)$	0.237***	0.973***	0.625**	0.738**	0.533**	0.380**	0.316*	0.269*	0.146
$Tms\,(\Omega_2)$	0.412***	1.514***	2.033***	2.226***	2.012***	1.880***	1.774***	1.743***	1.531***
$Tms\,(\Omega_3)$	0.580***	1.457***	1.973***	2.186***	2.062***	2.035***	1.969***	1.980***	1.800***
$Tms\,(\Omega_4)$	0.564***	1.437***	1.951***	2.164***	2.044***	2.020***	1.957***	1.968***	1.792***
$Tms\,(\Omega_0)$	0.074***	0.820***	0.557***	0.748**	0.544**	0.390**	0.358**	0.309*	0.127*

Notes: $\Omega_1 = [2\pi/12, \pi]$, $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_3 = [2\pi/96, 2\pi/36]$, $\Omega_4 = [0, 2\pi/96]$, $\Omega_0 = [0,\ \pi]$. The predictions obtained using the predictor $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Infl}\right)$ in the forecasting equation (14) are denoted as *Infl*(Ω_i) for $i = 0, 1, \ldots, 4$. *Tms*(Ω_i) stands for the same predictions using $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Tms}\right)$. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Clark and West (2006) (only reported for positive R^2_{OS} values).

Table 4: CER gains (Mean-Variance utility)

	Maturities								
	2	3	4	5	6	7	8	9	10
					$\gamma = 3$				
$Infl\,(\Omega_1)$	0.128	0.176	0.275	0.119	−0.029	−0.158	−0.206	−0.407	−0.517
$Infl\,(\Omega_2)$	0.401*	0.685**	0.806**	0.843**	0.904*	0.933*	1.194*	1.328**	1.450**
$Infl\,(\Omega_3)$	0.463*	0.749**	0.888**	0.935**	1.014*	1.061*	1.286*	1.467**	1.604**
$Infl\,(\Omega_4)$	0.457*	0.743**	0.887**	0.922**	1.017*	1.070*	1.285*	1.475**	1.622**
$Infl\,(\Omega_0)$	0.029	0.103	0.194	0.020	−0.151	−0.299	−0.318	−0.585	−0.670
$Tms\,(\Omega_1)$	0.433*	0.567*	0.496*	0.564*	0.819*	0.774	0.837	0.948	0.935
$Tms\,(\Omega_2)$	0.238	0.510*	0.699*	0.644	0.776	1.007*	1.285*	1.738**	1.888**
$Tms\,(\Omega_3)$	0.163	0.327	0.397	0.369	0.604	0.982	1.259*	1.605**	1.722**
$Tms\,(\Omega_4)$	0.156	0.317	0.391	0.352	0.586	0.966	1.241*	1.575**	1.688*
$Tms\,(\Omega_0)$	0.453*	0.579*	0.475	0.542	0.755	0.893*	1.088*	1.336**	1.374*
					$\gamma = 5$				
$Infl\,(\Omega_1)$	0.121	0.154	0.053	−0.099	−0.340	−0.439	−0.581	−0.748	−0.892
$Infl\,(\Omega_2)$	0.422*	0.653**	0.671**	0.635*	0.750*	0.907*	0.994*	0.738	0.668
$Infl\,(\Omega_3)$	0.478**	0.725**	0.745**	0.726*	0.842*	1.045**	1.129**	0.859	0.782
$Infl\,(\Omega_4)$	0.478**	0.721**	0.745**	0.741**	0.849*	1.068**	1.156**	0.888	0.811
$Infl\,(\Omega_0)$	0.079	0.116	−0.042	−0.186	−0.468	−0.582	−0.757	−0.898	−0.950
$Tms\,(\Omega_1)$	0.427*	0.489*	0.533*	0.688*	0.749*	0.839*	0.792	0.697	0.617
$Tms\,(\Omega_2)$	0.276	0.471*	0.492	0.684*	0.966*	1.442**	1.595**	1.664**	1.638**
$Tms\,(\Omega_3)$	0.158	0.232	0.271	0.506	0.742	1.174*	1.334*	1.544*	1.611*
$Tms\,(\Omega_4)$	0.150	0.227	0.263	0.493	0.725	1.146*	1.306*	1.527*	1.576*
$Tms\,(\Omega_0)$	0.427**	0.506**	0.467	0.718*	0.935**	1.064**	1.114**	1.063*	1.005
					$\gamma = 8$				
$Infl\,(\Omega_1)$	0.084	0.055	−0.093	−0.235	−0.489	−0.661	−0.707	−0.666	−0.700
$Infl\,(\Omega_2)$	0.401**	0.542**	0.443*	0.601**	0.701*	0.482	0.384	0.245	0.215
$Infl\,(\Omega_3)$	0.460**	0.601**	0.515*	0.706**	0.815**	0.596	0.476	0.303	0.274
$Infl\,(\Omega_4)$	0.462**	0.599**	0.525*	0.722**	0.835**	0.617	0.495	0.319	0.281
$Infl\,(\Omega_0)$	0.064	−0.019	−0.206	−0.381	−0.610	−0.746	−0.844	−0.803	−0.813
$Tms\,(\Omega_1)$	0.366**	0.433*	0.494*	0.632*	0.724*	0.573	0.444	0.248	0.162
$Tms\,(\Omega_2)$	0.260*	0.300	0.512*	0.910**	1.219**	1.201**	1.122*	0.866	0.645
$Tms\,(\Omega_3)$	0.130	0.096	0.308	0.664*	0.908*	1.082*	1.065	0.931	0.707
$Tms\,(\Omega_4)$	0.124	0.088	0.299	0.647*	0.890*	1.054*	1.041	0.912	0.690
$Tms\,(\Omega_0)$	0.372**	0.437*	0.573**	0.774**	0.918**	0.760	0.681	0.509	0.386

Notes: $\Omega_1 = [2\pi/12, \pi]$, $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_3 = [2\pi/96, 2\pi/36]$, $\Omega_4 = [0, 2\pi/96]$, $\Omega_0 = [0, \pi]$. The predictions obtained using the predictor $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Infl}\right)$ in the forecasting equation (14) are denoted as $Infl(\Omega_i)$ for $i = 0, 1, \ldots, 4$. $Tms\,(\Omega_i)$ stands for the same predictions using $\hat{F}_t\left(\Omega_i, \mathbf{z}^{Tms}\right)$. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Diebold and Mariano (1995) (only reported for positive CER gains).

Table 5: R^2_{OS} AND CER GAINS USING $\hat{F}\left(\Omega_4, \mathbf{z}^{Infl}\right)$ AND $\hat{F}\left(\Omega_2, \mathbf{z}^{Tms}\right)$

	Maturities								
	2	3	4	5	6	7	8	9	10
				Panel A: Out of sample R^2					
$Infl\,(\Omega_4)$	4.578***	3.990***	2.679***	2.170***	2.142***	1.987**	1.982**	1.725**	1.597**
$Tms\,(\Omega_2)$	0.412***	1.514***	2.033***	2.226***	2.012***	1.880***	1.774***	1.743***	1.531***
Both	5.934***	5.560***	4.495***	4.125***	4.184***	4.170***	4.056***	3.665***	3.354***
				Panel B: CER gains					
				Mean-Variance utility, $\gamma = 3$					
$Infl\,(\Omega_4)$	0.457*	0.743**	0.887**	0.922**	1.017*	1.070*	1.285*	1.475**	1.622**
$Tms\,(\Omega_2)$	0.238	0.510*	0.699*	0.644	0.776	1.007*	1.285*	1.738**	1.888**
Both	0.473*	0.717*	1.017**	1.229**	1.596**	2.064**	2.478**	2.779***	2.999***
				Mean-Variance utility, $\gamma = 5$					
$Infl\,(\Omega_4)$	0.478**	0.721**	0.745**	0.741**	0.849*	1.068**	1.156**	0.888	0.811
$Tms\,(\Omega_2)$	0.276	0.471*	0.492	0.684*	0.966*	1.442**	1.595**	1.664**	1.638**
Both	0.453*	0.649*	0.946**	1.247**	1.675**	2.185***	2.376***	2.397**	2.466**
				Mean-Variance utility, $\gamma = 8$					
$Infl\,(\Omega_4)$	0.462**	0.599**	0.525*	0.722**	0.835**	0.617	0.495	0.319	0.281
$Tms\,(\Omega_2)$	0.260*	0.300	0.512*	0.910**	1.219**	1.201**	1.122*	0.866	0.645
Both	0.433*	0.635**	0.870**	1.231**	1.603***	1.726**	1.684**	1.479*	1.370*

Notes: Forecasts labelled *Both* are obtained as in equation (18) for $i = 4$ and $j = 2$. In Panel A *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Clark and West (2006) (only reported for positive R^2_{OS} values). In Panel B *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Diebold and Mariano (1995) (only reported for positive CER gains).

Table 6: THE PREDICTIVE POWER OF THE UNSPANNED COMPONENT OF $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$: R^2_{OS}

	Maturities								
	2	3	4	5	6	7	8	9	10
$\mathbf{c}^{(1)}$	4.335***	4.098***	2.916***	2.342***	2.148***	1.919**	1.904**	1.663**	1.483**
$\mathbf{c}^{(2)}$	4.075***	3.848***	2.676***	2.153***	1.971***	1.724**	1.723**	1.503**	1.316**
$\mathbf{c}^{(3)}$	4.025***	3.916***	2.770***	2.205***	1.996***	1.681**	1.667**	1.473**	1.253**
$\mathbf{c}^{(4)}$	4.420***	3.754***	2.450***	2.082***	2.069***	1.944**	1.831**	1.515**	1.283**
$Infl\,(\Omega_4)$	4.578***	3.990***	2.679***	2.170***	2.142***	1.987**	1.982**	1.725**	1.597**

Notes: The R^2_{OS} reported in each line are obtained controlling for yield variation as described in Section 4.3, where the controls $\mathbf{c}^{(1)}$, $\mathbf{c}^{(2)}$, $\mathbf{c}^{(3)}$, $\mathbf{c}^{(4)}$ are the Cochrane-Piazzesi factor, 3 and 5 principal components of yields, and our term spread factor $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$, respectively. For readability, the R^2_{OS}s obtained with $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$ (rather than its unspanned component) are repeated here and denoted by $Infl\,(\Omega_4)$ as in Table 3. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Clark and West (2006) (only reported for positive R^2_{OS} values).

Table 7: Macroeconomic determinants of expected excess returns

	Maturities								
	2	3	4	5	6	7	8	9	10
					IP y-o-y growth				
$Infl\,(\Omega_4)$	−0.147***	−0.149***	−0.151***	−0.151***	−0.157***	−0.165***	−0.164***	−0.172***	−0.177***
$Tms\,(\Omega_2)$	−0.052	−0.054	−0.055	−0.056	−0.056	−0.059	−0.055	−0.060	−0.063
Both	−0.106**	−0.094**	−0.079*	−0.080*	−0.094**	−0.101**	−0.097**	−0.102**	−0.109**
					CFNAI C&H				
$Infl\,(\Omega_4)$	−0.061	−0.089*	−0.133***	−0.149***	−0.132***	−0.144***	−0.141***	−0.167***	−0.176***
$Tms\,(\Omega_2)$	−0.093**	−0.106**	−0.125***	−0.142***	−0.136***	−0.154***	−0.147***	−0.167***	−0.177***
Both	−0.054	−0.073	−0.100**	−0.118**	−0.114**	−0.131***	−0.128***	−0.146***	−0.158***
					MCSI				
$Infl\,(\Omega_4)$	−0.150***	−0.158***	−0.168***	−0.170***	−0.165***	−0.170***	−0.164***	−0.173***	−0.177***
$Tms\,(\Omega_2)$	−0.256***	−0.267***	−0.277***	−0.287***	−0.283***	−0.291***	−0.284***	−0.295***	−0.301***
Both	−0.266***	−0.300***	−0.331***	−0.343***	−0.338***	−0.350***	−0.347***	−0.361***	−0.369***

Notes: Correlation between expected returns and macroeconomic cyclical indicators. IP y-o-y growth stands is the year-on-year industrial production growth, CFNAI C&H is the Chicago Fed National Activity sub-index on consumption and housing, MCSI is the Michigan consumer sentiment index. $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_4 = [0, 2\pi/96]$. Forecasts labelled *Both* are obtained as in equation (18) for $i = 4$ and $j = 2$. *, **, *** denote statistical significance at 10, 5, 1 percent level.

Table 8: Out of sample R^2 in expansions and recessions

	Maturities								
	2	3	4	5	6	7	8	9	10
					Expansions				
$Infl\,(\Omega_4)$	−1.179	0.350**	0.202*	−0.011	−0.357	−0.708	−0.642	−0.556	−0.628
$Tms\,(\Omega_2)$	0.231***	1.492***	2.023***	2.024***	1.790***	1.557***	1.566***	1.479***	1.290***
Both	0.162***	1.940***	2.131***	2.035***	1.797***	1.414***	1.439***	1.444***	1.229***
					Recessions				
$Infl\,(\Omega_4)$	20.969**	17.282***	12.847***	11.972***	14.482***	14.387**	13.494**	11.632**	10.859**
$Tms\,(\Omega_2)$	0.928*	1.599*	2.073*	3.132*	3.095*	3.335	2.634	2.829	2.464
Both	22.355**	18.774**	14.205**	13.512**	15.949**	16.805**	15.455**	13.222**	12.097**

Notes: $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_4 = [0, 2\pi/96]$. Forecasts labelled *Both* are obtained as in equation (18) for $i = 4$ and $j = 2$. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Clark and West (2006) (only reported for positive R^2_{OS} values).

Table 9: CER gains in expansions and recessions

	Maturities								
	2	3	4	5	6	7	8	9	10
					Expansions				
$Infl\,(\Omega_4)$	0.260	0.425*	0.362	0.350	0.410	0.584	0.640	0.386	0.286
$Tms\,(\Omega_2)$	0.302	0.491*	0.478	0.646	0.932*	1.394**	1.558**	1.663**	1.730**
Both	0.207	0.328	0.536	0.813*	1.168*	1.656**	1.892**	2.100**	2.253**
					Recessions				
$Infl\,(\Omega_4)$	2.672*	3.708*	4.608**	4.686**	5.265**	5.941**	6.352*	5.942*	6.096*
$Tms\,(\Omega_2)$	0.017	0.279	0.647	1.101	1.345	1.896	1.832	1.468	0.426
Both	2.932*	3.877*	5.123**	5.672**	6.812**	7.474**	7.120*	5.191	4.332

Notes: $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_4 = [0, 2\pi/96]$. Forecasts labelled *Both* are obtained as in equation (18) for $i = 4$ and $j = 2$. CER gains are calculated as in the economic evaluation exercise described in Section 4.2 under mean-variance prefences and with $\gamma = 5$. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Diebold and Mariano (1995) (only reported for positive CER gains).

Figures

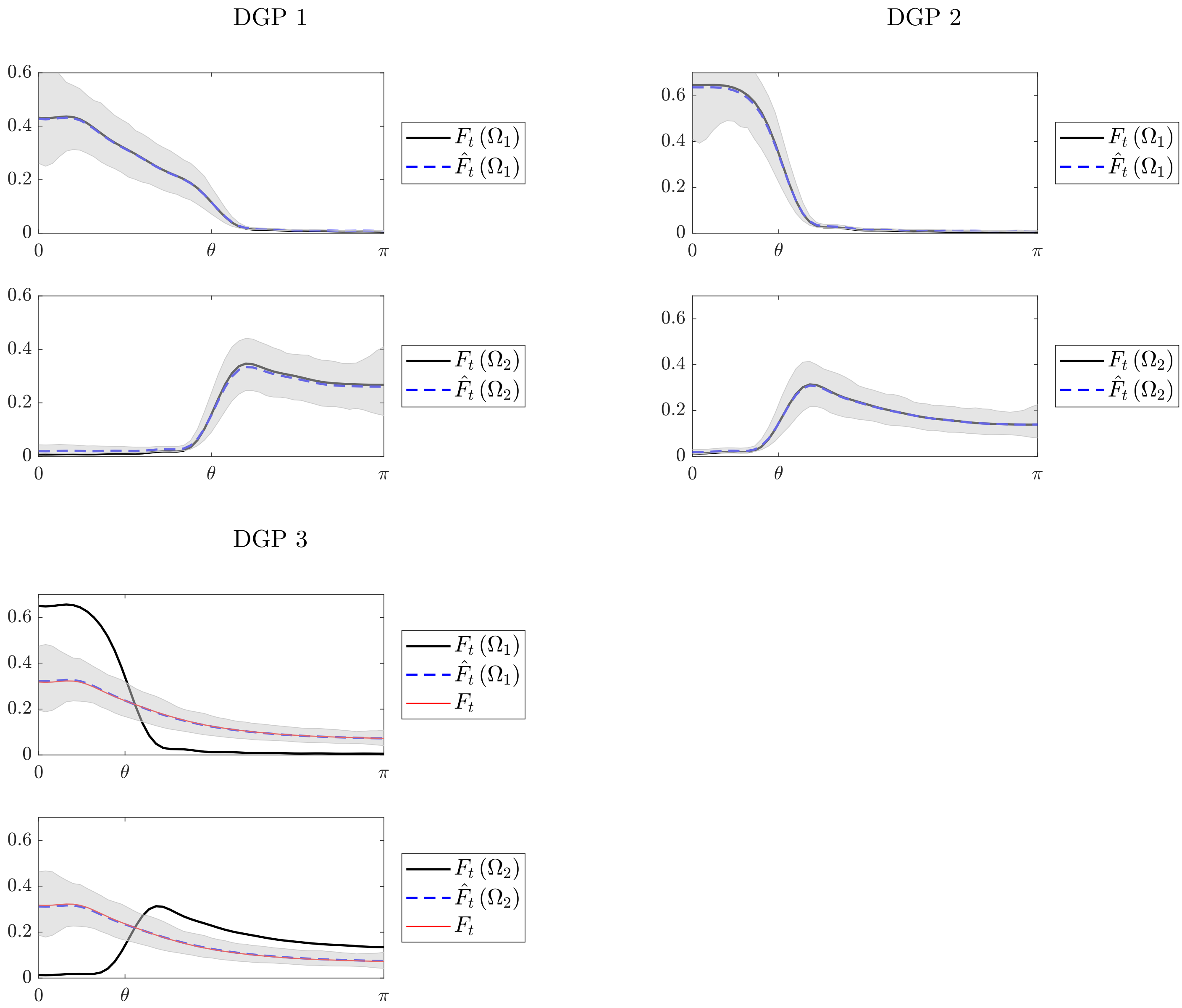


Figure 1: Simulation: spectral density of estimated factors

Notes: Simulation exercise for the DGPs described in Section 2.2.2 with $(T, N) = (200, 200)$ and $r = 1$. Spectral densities are estimated using a lag-window estimator (9). The solid lines are the spectra obtained with the unfeasible lag-window estimator that uses true factors, the dashed lines are instead obtained using the factors estimated via BSPCs. Shaded areas denote 95% confidence bands around the feasible BSPC estimates.

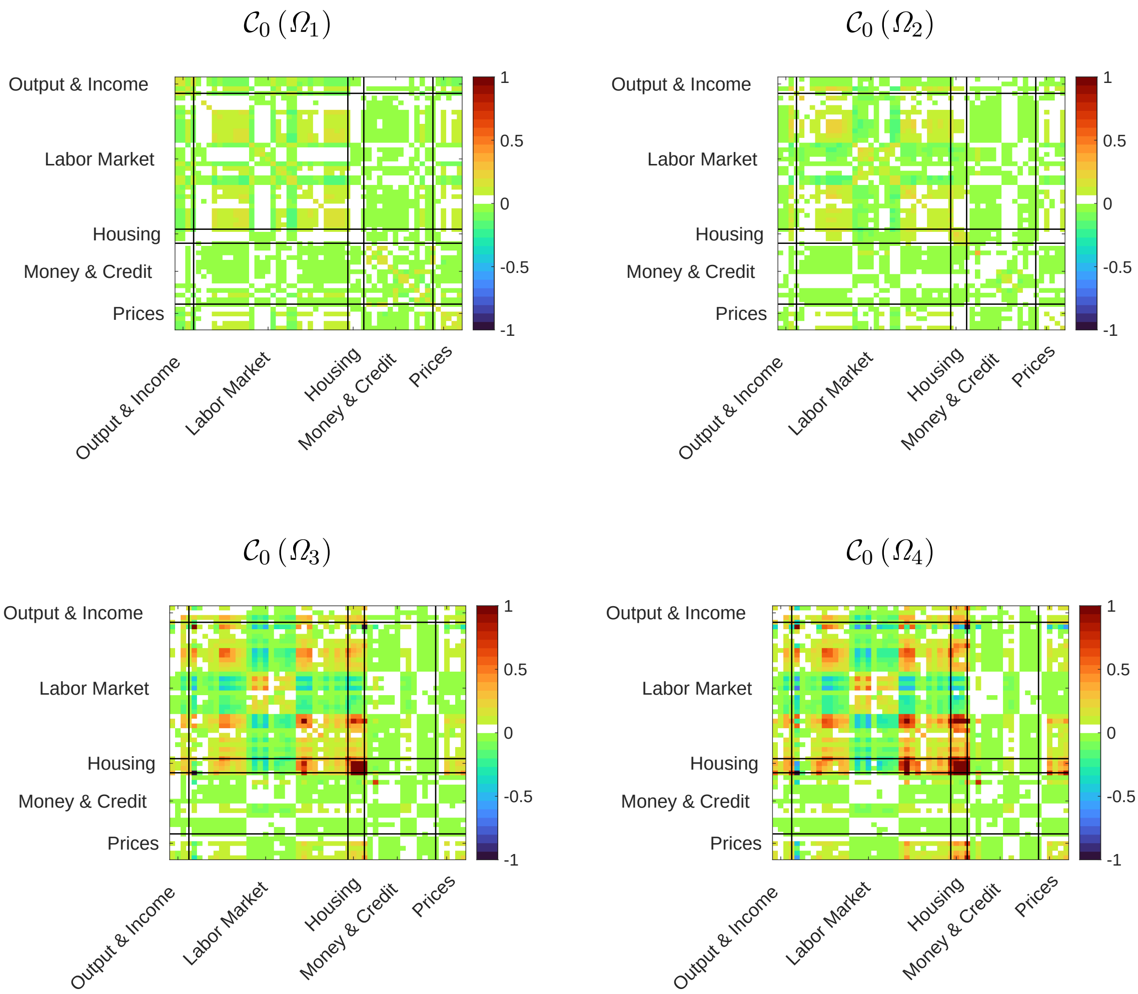


Figure 2: ALFRED DATASET: COVARIANCE MATRIX DECOMPOSITION BY ITS CYCLICAL COMPONENTS

Notes: Cycle length are: up to 1 year (Ω_1); between 1 and 3 years (Ω_2); between 3 and 8 years (Ω_3); 8+ years (Ω_4). For the generic band $\Omega = [\underline{\omega}, \overline{\omega}]$, we consider the normalised band spectrum covariance $\mathcal{C}_0\left(\Omega\right) = 0.5\left(\overline{\omega} - \underline{\omega}\right)^{-1} \mathbf{C}_0\left(\Omega\right)$.

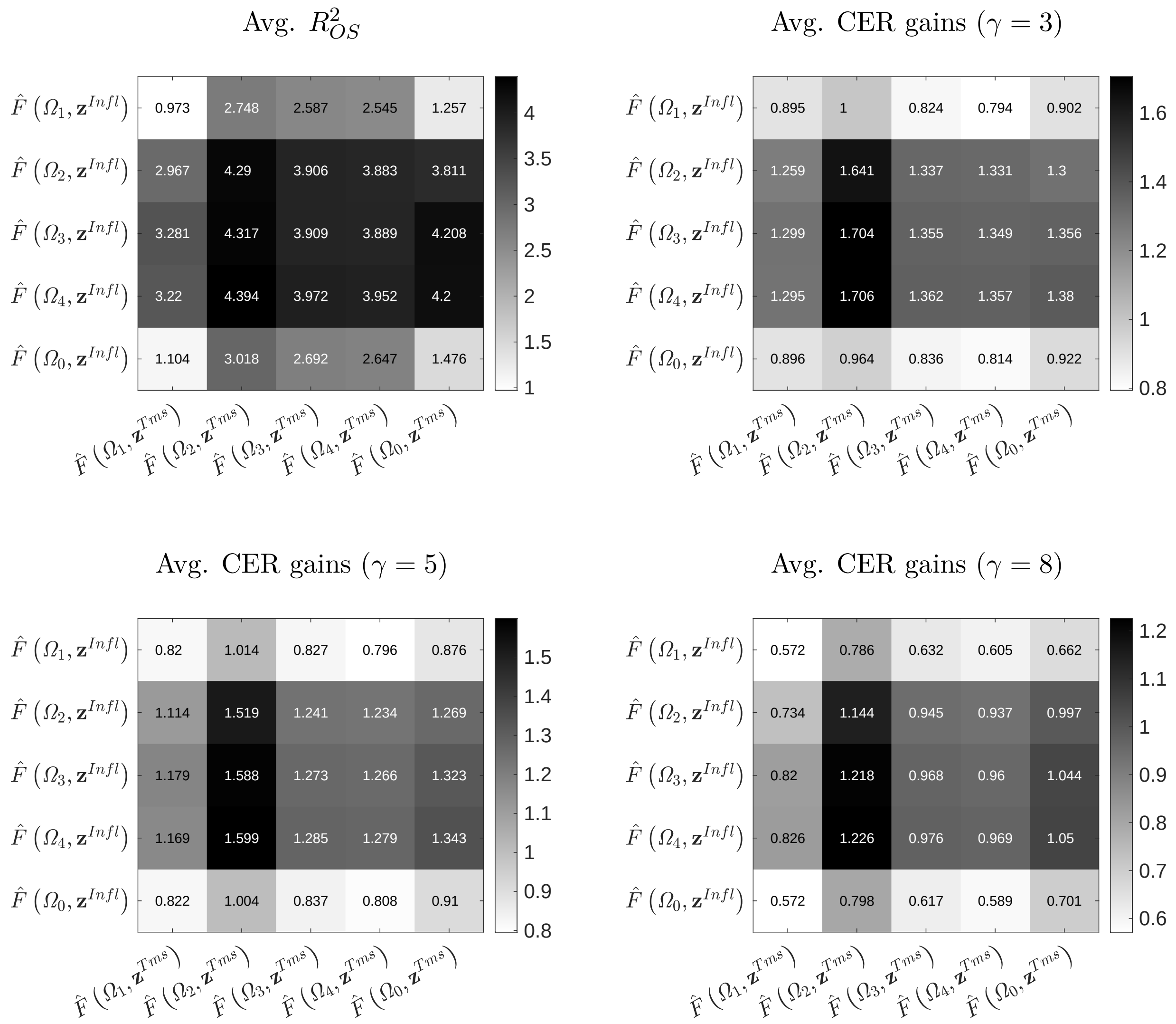
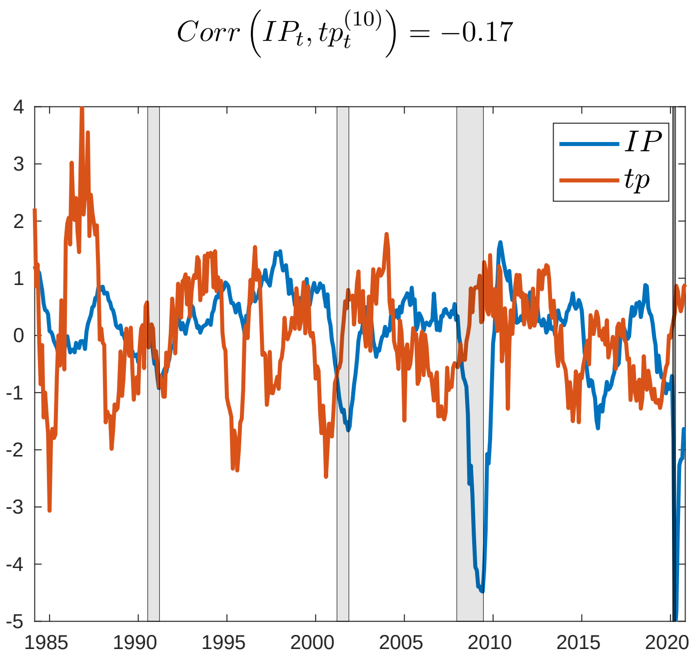


Figure 3: Average R^2_{OS} and CER gains using all combinations of two predictors

Notes: Average out-of-sample R^2 and CER gains under mean-variance preferences across maturities corresponding to the predictions obtained as in equation (18) for any $i, j = 0, 1, \ldots, 4$.



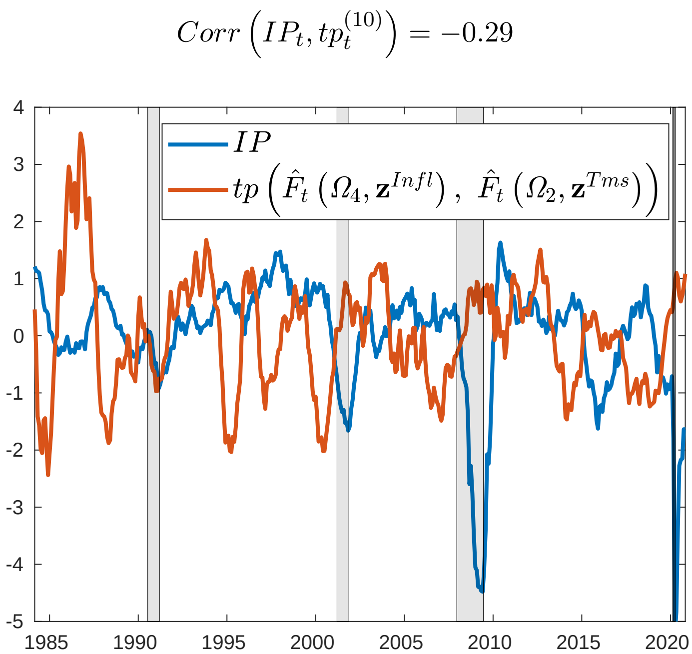


Figure 4: Cyclical properties of the term premium

Notes: The term premium is estimated as in equation (19). In the top plot, only yields are used to predict excess bond returns. In the bottom plot the expected excess bond returns are obtained using our macroeconomic predictors $\hat{F}_t\left(\Omega_4, \mathbf{z}^{Infl}\right)$ and $\hat{F}_t\left(\Omega_2, \mathbf{z}^{Tms}\right)$.

Appendix

A. Frequency-specific factor estimation

In this section we establish the consistent estimation of the space spanned by frequency-specific factors which motivates the use of estimated factors in our predictive regressions as if such factors were observed.

For $j = 1, 2$, define the N-dimensional vectors $\mathbf{X}_t(\Omega_j) = \int_{\omega\in\Omega_j} \mathcal{X}_\omega e^{\iota\omega t} d\omega$ and $\mathbf{e}_t(\Omega_j) = \int_{\omega\in\Omega_j} \mathcal{E}_\omega e^{\iota\omega t} d\omega$, the $T \times N$ matrices $\mathbf{X}(\Omega_j) = (\mathbf{X}_1(\Omega_j), \mathbf{X}_2(\Omega_j), \cdots, \mathbf{X}_T(\Omega_j))'$ and $\mathbf{E}(\Omega_j) = (\mathbf{e}_1(\Omega_j), \mathbf{e}_2(\Omega_j), \cdots, \mathbf{e}_T(\Omega_j))'$ with generic entries $x_{it}(\Omega_j)$ and $e_{it}(\Omega_j)$, respectively, and the T-dimensional vectors $\mathbf{e}_i = (e_{i1}, e_{i2}, \cdots, e_{iT})'$, $\mathbf{e}_i(\Omega_j) = (e_{i1}(\Omega_j), e_{i2}(\Omega_j), \cdots, e_{iT}(\Omega_j))'$. We will use the singular value decomposition

$$
\begin{aligned}
(NT)^{-1/2} \mathbf{X}(\Omega_j) &= V_{NT}(\Omega_j) D_{NT}(\Omega_j) U_{NT}(\Omega_j)' \\
&= V_{NT,r}(\Omega_j) D_{NT,r}(\Omega_j) U_{NT,r}(\Omega_j)' + V_{NT,N-r}(\Omega_j) D_{NT,N-r}(\Omega_j) U_{NT,N-r}(\Omega_j)'
\end{aligned}
$$

where the diagonal entries of $D_{NT}(\Omega_j)$ are sorted in decreasing order. Finally, $\|A\| = \sqrt{tr(AA')}$.

Assumption 1. *For $i = 1, 2$*

(i) $E(e_{it}|\Lambda_1, \Lambda_2, \mathbf{F}_t(\Omega_1), \mathbf{F}_t(\Omega_2)) = 0$

(ii) It exists $M < \infty$ such that

- *(a)* $E\left(N^{-1/2} \sum_{i=1}^{N} (e_{it}e_{is} - E(e_{it}e_{is}))\right)^2 \leq M$*;*
- *(b)* $T^{-1} \sum_{t=1}^{T} \sum_{s=1}^{T} |E(e_{it}e_{is})| \leq M$*, for all i;*
- *(c)* $N^{-1}T^{-1/2} \|\mathbf{e}_t'\mathbf{E}'\| = O_p\left(\min\left(N^{-1/2}, T^{-1/2}\right)\right)$*, for all i;*
- *(d)* $T^{-1}N^{-1/2} \|\mathbf{e}_i'\mathbf{E}\| = O_p\left(\min\left(N^{-1/2}, T^{-1/2}\right)\right)$*, for all t.*

Assumption 2. *For $j = 1, 2$*

(i) $lim_{N\to\infty}\ N^{-1}\Lambda_j'\Lambda_j = \mathbf{C}_{\Lambda,j}$

(ii) $plim_{T\to\infty}\ T^{-1}\mathbf{F}(\Omega_j)' \mathbf{F}(\Omega_j) = \mathbf{C}_F(\Omega_j)$

where $\mathbf{C}_{\Lambda,j}$ and $\mathbf{C}_F(\Omega_j)$ are positive definite with distinct eigenvalues.

Assumption 3. *For $j = 1, 2$*

(i) $E\left\|N^{-1/2} \sum_i \Lambda_j e_{it}(\Omega_j)\right\|^2 \leq M$ *and* $(NT)^{-1} \mathbf{e}_t(\Omega_j)' \mathbf{E}(\Omega_j)' \mathbf{F}(\Omega_j)' = O_p(\min(N^{-1}, T^{-1}))$, *for each t;*

(ii) $E\left\|T^{-1/2} \sum_{t=1}^{T} \mathbf{F}_t(\Omega_j) e_{it}(\Omega_j)\right\|^2 \leq M$ *and* $(NT)^{-1} \mathbf{e}_i(\Omega_j)' \mathbf{E}(\Omega_j) \Lambda_j = O_p(\min(N^{-1}, T^{-1}))$, *for each i.*

Assumption 1 corresponds to Assumption A1 of Bai and Ng (2020), while Assumptions 2, 3 merely readapt their Assumptions A2, A3 to our context with frequency-specific factors. Under these assumptions and following the same steps as in Proposition 1 of Bai and Ng (2020), it is straightforward to obtain that

$$\begin{aligned}
&(NT)^{-1}\,\mathbf{X}(\Omega_j)\,\mathbf{X}(\Omega_j)'\,\hat{\mathbf{F}}(\Omega_j) = \hat{\mathbf{F}}(\Omega_j)\,D_{NT,r}(\Omega_j) \\
&= (NT)^{-1}\left(\mathbf{F}(\Omega_j)\,\Lambda_j'\Lambda_j\mathbf{F}(\Omega_j)' + \mathbf{F}(\Omega_j)\,\Lambda_j'\mathbf{E}(\Omega_j)' + \mathbf{E}(\Omega_j)\,\Lambda_j\mathbf{F}(\Omega_j)' + \mathbf{E}(\Omega_j)\,\mathbf{E}(\Omega_j)'\right)\hat{\mathbf{F}}(\Omega_j)
\end{aligned}$$

Defining the rotation matrix $H_{NT}(\Omega_j) = \left(\frac{\Lambda_j'\Lambda_j}{N}\right)\left(\frac{\mathbf{F}(\Omega_j)\hat{\mathbf{F}}(\Omega_j)}{T}\right)D_{NT,r}(\Omega_j)^{-1}$, we have

$$\begin{aligned}
T^{-1}\left\|\hat{\mathbf{F}}(\Omega_j) - \mathbf{F}(\Omega_j)\,H_{NT}(\Omega_j)\right\|^2 &\le \frac{2}{T}\left\|\frac{\mathbf{E}(\Omega_j)\,\Lambda_j}{N}\right\|^2 \frac{\|\mathbf{F}(\Omega_j)\|^2}{T}\left\|\hat{\mathbf{F}}(\Omega_j)\right\|^2 T^{-1}\left\|D_{NT,r}(\Omega_j)^{-1}\right\|^2 + \\
&+\left\|\frac{\mathbf{E}(\Omega_j)\,\mathbf{E}(\Omega_j)'}{NT}\right\|^2\left\|\hat{\mathbf{F}}(\Omega_j)\right\|^2 T^{-1}\left\|D_{NT,r}(\Omega_j)^{-1}\right\|^2 \\
&= A + B
\end{aligned}$$

A is $O_p(N^{-1})$ because $T^{-1}\left\|\frac{\mathbf{E}(\Omega_j)\Lambda_j}{N}\right\|^2$ is $O_p(N^{-1})$ by Assumption 3, $\|\mathbf{F}(\Omega_j)\|^2 T^{-1}$ is $O_p(1)$ by Assumption 2, $\left\|\hat{\mathbf{F}}(\Omega_j)\right\|^2 T^{-1} = r$ because $\hat{\mathbf{F}}(\Omega_j) = \sqrt{T}\mathbf{X}V_{NT,r}(\Omega_j)$ and the columns of $V_{NT,r}(\Omega_j)$ are unit-length, and $\left\|D_{NT,r}(\Omega_j)^{-1}\right\|^2$ is $O_p(1)$ since by Assumption 2 the largest r eigenvalues of $T^{-1}\mathbf{X}(\Omega_j)\,\mathbf{X}(\Omega_j)'$ are $O_p(N)$.

B is $O_p(T^{-1})$ because, under Assumption 1, $\left\|\frac{\mathbf{E}(\Omega_j)\mathbf{E}(\Omega_j)'}{NT}\right\|^2 \le \left\|\frac{\mathbf{E}\mathbf{E}'}{NT}\right\|^2 = O_p(\min(N^{-1}, T^{-1}))$ where the last equality is established by Lemma 1 of Bai and Ng (2020).

Hence, $T^{-1}\left\|\hat{\mathbf{F}}(\Omega_j) - \mathbf{F}(\Omega_j)\,H_{NT}(\Omega_j)\right\|^2$ is $O_p(\min(N^{-1}, T^{-1}))$.

B. Real-time macroeconomic data

Table B.1: ALFRED data

	Mnemonic	Description	Tcode
1	AWHMAN	Avg Weekly Hours of Production and Nonsupervisory Employees: Manufacturing	1
2	AWHNONAG	Avg Weekly Hours Of Production And Nonsupervisory Employees: Total private	2
3	AWOTMAN	Avg Weekly Overtime Hours of Production and Nonsupervisory Employees: Manufacturing	2
4	CE16OV	Civilian Employment	5
5	CLF16OV	Civilian Labor Force	5
6	CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items	6
7	CURRDD	Currency Component of M1 Plus Demand Deposits	6
8	CURRSL	Currency Component of M1	5
9	DEMDEPSL	Demand Deposits at Commercial Banks	6
10	DMANEMP	All Employees: Durable goods	5
11	DSPI	Disposable Personal Income	5
12	DSPIC96	Real Disposable Personal Income	5
13	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	4
14	HOUST1F	Privately Owned Housing Starts: 1-Unit Structures	4
15	HOUST2F	Housing Starts: 2-4 Units	4
16	INDPRO	Industrial Production Index	5
17	M1SL	M1 Money Stock	6
18	MANEMP	All Employees: Manufacturing	5
19	NDMANEMP	All Employees: Nondurable goods	5
20	OCDSL	Other Checkable Deposits	6
21	PAYEMS	All Employees: Total nonfarm	5
22	PCE	Personal Consumption Expenditures	5
23	PCEDG	Personal Consumption Expenditures: Durable Goods	5
24	PCEND	Personal Consumption Expenditures: Nondurable Goods	5
25	PCES	Personal Consumption Expenditures: Services	5
26	PI	Personal Income	5
27	SAVINGSL	Savings Deposits - Total	6
28	SRVPRD	All Employees: Service-Providing Industries	5
29	STDCBSL	Small Time Deposits at Commercial Banks	6
30	STDSL	Small Time Deposits - Total	6
31	STDTI	Small Time Deposits at Thrift Institutions	6
32	SVGCBSL	Savings Deposits at Commercial Banks	6
33	SVGTI	Savings Deposits at Thrift Institutions	6
34	SVSTCBSL	Savings and Small Time Deposits at Commercial Banks	6
35	TCDSL	Total Checkable Deposits	6
36	UEMP5TO14	Civilians Unemployed for 5-14 Weeks	5
37	UEMP15OV	Civilians Unemployed - 15 Weeks & Over	5
38	UEMP15T26	Civilians Unemployed for 15-26 Weeks	5
39	UEMP27OV	Civilians Unemployed for 27 Weeks and Over	5
40	UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	5
41	UEMPMEAN	Average (Mean) Duration of Unemployment	2
42	UEMPMED	Median Duration of Unemployment	2
43	UNEMPLOY	Unemployed	5
44	UNRATE	Civilian Unemployment Rate	2
45	USCONS	All Employees: Construction	5
46	USFIRE	All Employees: Financial Activities	5
47	USGOOD	All Employees: Goods-Producing Industries	5
48	USGOVT	All Employees: Government	5
49	USMINE	All Employees: Mining and logging	5
50	USPRIV	All Employees: Total Private Industries	5
51	USSERV	All Employees: Other Services	5
52	USTPU	All Employees: Trade, Transportation & Utilities	5
53	USTRADE	All Employees: Retail Trade	5
54	USWTRADE	All Employees: Wholesale Trade	5

Notes: Tcode indicates the transformation adopted to achieve stationarity and is as follows. Letting $\tilde{x}_{it}$ be a raw variable and x_{it} its stationary transformation, we consider one of the following six transformation codes. 1: $x_{it} = \tilde{x}_{it}$; 2: $x_{it} = (1-L)\,\tilde{x}_{it}$; 3: $x_{it} = (1-L)^2\,\tilde{x}_{it}$; 4: $x_{it} = \ln(\tilde{x}_{it})$; 5: $x_{it} = (1-L)\ln\tilde{x}_{it}$; 6: $x_{it} = (1-L)^2\ln\tilde{x}_{it}$.

C. Additional empirical results

Table C.1: Out-of-sample R^2 using SUFF and sSUFF

	Maturities								
	2	3	4	5	6	7	8	9	10
SUFF	−6.899	−6.089	−5.515	−4.184	−5.078	−4.859	−5.419	−3.765	−5.110
sSUFF	−5.780	−5.276	−5.954	−5.220	−5.121	−5.165	−4.853	−3.593	−4.769
SUFF $(Infl)$	−3.848	−1.930	−1.121	−0.838	−0.393	−0.340	−0.703	−0.029	0.544**
sSUFF $(Infl)$	−7.865	−6.756	−7.163	−5.662	−5.526	−4.775	−4.734	−4.273	−5.010
SUFF (Tms)	−3.695	−2.732	−1.724	−3.037	−0.463	−0.878	−1.485	−3.437	−4.681
sSUFF (Tms)	−3.020	−0.943	−1.804	−1.261	−2.993	−8.760	−5.543	−4.936	−4.418
SUFF $(Infl, Tms)$	−6.158	−4.290	−4.965	−2.847	−2.294	0.166*	0.106*	−0.756	1.352**
sSUFF $(Infl, Tms)$	−3.255	−3.698	−3.709	−3.759	−4.434	−4.957	−5.629	−5.407	−5.514

Notes: SUFF and sSUFF are performed as described in Huang et al. (2023) and extracting 6 latent factors and 2 predictive indices. Where additional regressors are indicated in brackets the predictors are first projected onto the sieve space of such additional regressors as in Fan et al. (2017, 2021). *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Clark and West (2006) (only reported for positive R^2_{OS} values).

Table C.2: CER GAINS (POWER UTILITY)

	Maturities								
	2	3	4	5	6	7	8	9	10
					$\gamma = 3$				
$Infl\,(\Omega_1)$	0.126	0.166	0.261	0.201	0.061	−0.085	−0.132	−0.186	−0.260
$Infl\,(\Omega_2)$	0.387*	0.640*	0.804**	0.876**	0.892*	0.983*	1.174*	1.400**	1.571**
$Infl\,(\Omega_3)$	0.454*	0.709**	0.883**	0.960**	0.994*	1.076*	1.267*	1.511**	1.709**
$Infl\,(\Omega_4)$	0.449*	0.706**	0.881**	0.948**	0.991*	1.082*	1.273*	1.511**	1.700**
$Infl\,(\Omega_0)$	0.029	0.073	0.213	0.088	−0.034	−0.192	−0.228	−0.331	−0.410
$Tms\,(\Omega_1)$	0.395*	0.551*	0.498*	0.536	0.712	0.744	0.763	0.921	0.938
$Tms\,(\Omega_2)$	0.209	0.463*	0.725*	0.658	0.775	0.950	1.145*	1.647**	1.810**
$Tms\,(\Omega_3)$	0.139	0.287	0.415	0.386	0.542	0.879	1.133	1.580**	1.683**
$Tms\,(\Omega_4)$	0.129	0.271	0.414	0.378	0.525	0.862	1.109	1.552*	1.651*
$Tms\,(\Omega_0)$	0.437*	0.554*	0.490*	0.512	0.676	0.823	0.923	1.266**	1.277*
Both	0.440*	0.670*	0.990**	1.167**	1.543**	1.924**	2.308**	2.713**	2.965***
					$\gamma = 5$				
$Infl\,(\Omega_1)$	0.116	0.167	0.092	−0.050	−0.291	−0.310	−0.456	−0.612	−0.689
$Infl\,(\Omega_2)$	0.404*	0.646**	0.679**	0.683*	0.729*	0.943*	1.059*	0.907*	0.876
$Infl\,(\Omega_3)$	0.459**	0.712**	0.737**	0.750**	0.822*	1.040**	1.192**	1.031*	0.989*
$Infl\,(\Omega_4)$	0.460**	0.710**	0.732**	0.758**	0.823*	1.056**	1.213**	1.053*	1.013*
$Infl\,(\Omega_0)$	0.072	0.116	0.002	−0.141	−0.416	−0.455	−0.617	−0.781	−0.770
$Tms\,(\Omega_1)$	0.417*	0.497*	0.484*	0.675*	0.687*	0.822*	0.797	0.731	0.702
$Tms\,(\Omega_2)$	0.259	0.467*	0.490	0.642*	0.858*	1.361**	1.540**	1.713**	1.766**
$Tms\,(\Omega_3)$	0.153	0.228	0.246	0.461	0.659	1.103*	1.262*	1.537*	1.663*
$Tms\,(\Omega_4)$	0.144	0.221	0.239	0.452	0.645	1.067*	1.231*	1.518*	1.641*
$Tms\,(\Omega_0)$	0.429**	0.509**	0.446	0.660*	0.850*	1.055**	1.100**	1.068*	1.045
Both	0.437*	0.629*	0.900**	1.187**	1.569**	2.142***	2.356***	2.448***	2.596***
					$\gamma = 8$				
$Infl\,(\Omega_1)$	0.085	0.074	−0.050	−0.213	−0.451	−0.607	−0.666	−0.594	−0.632
$Infl\,(\Omega_2)$	0.394**	0.538**	0.462*	0.585**	0.722*	0.559	0.474	0.399	0.420
$Infl\,(\Omega_3)$	0.451**	0.592**	0.538*	0.681**	0.840**	0.662	0.574	0.472	0.466
$Infl\,(\Omega_4)$	0.452**	0.591**	0.548**	0.695**	0.860**	0.683	0.593	0.487	0.469
$Infl\,(\Omega_0)$	0.066	0.002	−0.150	−0.340	−0.570	−0.693	−0.788	−0.711	−0.713
$Tms\,(\Omega_1)$	0.359**	0.415*	0.507*	0.612**	0.704*	0.578	0.470	0.324	0.242
$Tms\,(\Omega_2)$	0.249*	0.299	0.494*	0.847**	1.168**	1.214**	1.146*	0.964	0.766
$Tms\,(\Omega_3)$	0.129	0.098	0.303	0.600*	0.857*	1.052*	1.062*	1.018	0.824
$Tms\,(\Omega_4)$	0.122	0.088	0.296	0.583	0.838*	1.020*	1.034	0.998	0.804
$Tms\,(\Omega_0)$	0.368**	0.423*	0.559**	0.729**	0.890**	0.776*	0.698	0.558	0.438
Both	0.420*	0.603**	0.855**	1.168**	1.553***	1.719**	1.702**	1.554**	1.487*

Notes: $\Omega_1 = [2\pi/12, \pi]$, $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_3 = [2\pi/96, 2\pi/36]$, $\Omega_4 = [0, 2\pi/96]$, $\Omega_0 = [0, \pi]$. *Both* forecasts are obtained as in equation (18) for $i = 4$ and $j = 2$. *, **, *** denote statistical significance at 10, 5, 1 percent level using the test of Diebold and Mariano (1995) (only reported for positive CER gains).

Table C.3: Sharpe ratios in expansions and recessions

	Maturities								
	2	3	4	5	6	7	8	9	10
	Expansions								
$Infl\,(\Omega_4)$	0.227	0.218	0.189	0.172	0.166	0.153	0.151	0.127	0.122
$Tms\,(\Omega_2)$	0.221	0.212	0.190	0.184	0.185	0.182	0.181	0.170	0.170
Both	0.214	0.202	0.196	0.193	0.196	0.191	0.192	0.184	0.186
	Recessions								
$Infl\,(\Omega_4)$	0.684	0.652	0.574	0.521	0.499	0.428	0.385	0.334	0.315
$Tms\,(\Omega_2)$	0.621	0.581	0.470	0.408	0.388	0.332	0.285	0.240	0.208
Both	0.711	0.664	0.578	0.525	0.514	0.452	0.397	0.328	0.290

Notes: $\Omega_2 = [2\pi/36, 2\pi/12]$, $\Omega_4 = [0, 2\pi/96]$. *Both* forecasts are obtained as in equation (18) for $i = 4$ and $j = 2$. Sharpe ratios are calculated from portfolio returns obtained as in the economic evaluation exercise described in Section 4.2 under mean-variance prefences and with $\gamma = 5$.