Safe Distance to Systemic Risk*

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Preliminary version

Abstract

In this paper, we propose a new systemic risk indicator to measure the distance to the extreme losses of a financial system. Our indicator is based on cross-sectional concomitant VaR exceptions (Co-Exceptions) observed at a daily frequency, which are then converted into a weekly time series with only the maximum values to apply extreme value models. A set of 95 large U.S. financial institutions is used to run the empirical analysis over the last 20 years to check the real-time ability of our framework to predict significant financial crises, such as the Great Financial Crisis of 2008, the sovereign debt crisis of 2010 or the COVID lockdowns of 2020. Our systemic risk indicator identifies accurately this surge in systemic risk and provides additional information compared to the VIX indicator or the Value-at-Risk of the market. Finally, we show that this new measure of financial instability is explained by macroeconomic variables, such as industrial production and unemployment which have a positive impact, whereas the consumer price index, interest rate, and federal funds rate have a negative impact.

Keywords: Systemic risk; Extreme value theory; VaR Co-Exceptions; Financial crises

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1 Introduction

Keep safe distance to financial turmoils is not an easy task and financial crises in recent years underscore the urgent need for effective financial systemic risk measurement. These events reveals the limitations for the traditional systemic risk models to assess the extreme events. Beyond the necessity of developing risk measures to identify systemically important banks, many systemic risk measures capturing risk at a global level have been proposed since 2008. Indeed, it is necessary to track the evolution of systemic risk at an aggregate level over time to rely on a reliable thermometer and thus trigger regulatory measures to reduce systemic risk when necessary. Brownlees and Engle (2016) propose a bottom-up approach by aggregating individual banks' SRISK at a national or even supranational level. Conversely, the ECB relies on the top-down approach introduced by Hollo, Kremer, and Lo Duca (2012) with the Composite Indicator of Systemic Stress (CISS). The CISS builds on fundamental portfolio theory by aggregating five market-specific sub-indices, created from a total of 15 individual measures of financial stress.

Most of the systemic risk measures tend to focus on individual financial institutions' risk rather than the entire financial system's stability (Ellis, Sharma, and Brzeszczyński, 2022). Aiming to contribute to this literature on aggregated systemic risk, we propose introducing the concept of a safety distance to maintain in order to ensure financial stability. For this, we rely on the idea introduced by Caporin, Kolokolov, and Renò (2017) in seeking to quantify the simultaneous occurrence of jumps (co-jumps) in several stocks. These joint jumps can be associated with major financial news, triggering short-term predictability of stock returns, and determining a persistent increase (decrease) in variances and correlations of stocks when accompanied by bad (good) news. Thus, rare and spectacular multiple jumps can be interpreted as systemic co-jumps in Das and Uppal (2004). Instead of working at high frequency and on assets with significant trading volumes, we work at a daily frequency using the database proposed by Brownlees and Engle (2016) but extending from 2000 to 2023. We dynamically calculate daily VaR forecasts over the entire period for each financial asset and compare them to realized returns. Each day, we are thus able to count the simultaneous number of VaR exceptions, and these days correspond well to significant events in the financial markets, see Table 2. Therefore, to model extreme risks, we transform this daily series into a weekly series containing the maximum number of simultaneous VaR exceptions observed each week. In line with Hoga (2023), who emphasizes the importance of forecasting systemic risk measures predictive of financial crises and declines in real activity, our approach integrates these forecasts into a broader financial stability analysis. An extreme value model (GEV) is then estimated on a rolling two-year window to recover the return level and the probability of return. Gavronski and Ziegelmann (2021) also use extreme value theory in their development of the Financial System Dependence Index (FSDI), further validating the relevance of this approach in systemic risk measurement.

The crucial outputs of the GEV are the return level and the return probability, the return level refers to the magnitude of extreme loss that should be observed in the financial market and that should occur once over a given period. This allows for measuring the intensity of extreme events in the market. The return probability is the probability that an event of a certain magnitude will occur over a given period. The return level could be seen as the expected number of simultaneous exceptions that should be observed over a certain time horizon. When the return level increases, it means that financial instability is increasing, thus reducing the safety distance to the potential occurrence of a systemic event. This predicted return level effectively anticipates the occurrence of major financial events, such as the 2008 financial crisis, the 2010 European sovereign debt crisis, and the COVID period. We observe a peak in mid-2016 in our indicator but not in the VIX nor the VaR of the market caused by the crisis with the Deutsche Bank, this finding aligns with the work of Moratis and Sakellaris (2021), also in the 2016 Financial Stability Report of the International Monetary Fund, pointed out that Deutsche Bank was the largest net contributor to global systemic risk, with its ISR score reaching a peak in July 2016.

Finally, we explain (via regression) the return level with macroeconomic variables like interest rate, unemployment rate, consumer price index, and so on. For the variables particularly those reflecting monetary policy, inflation, and systemic stress, are paramount in determining the financial system's risk level as captured by our indicator. This helps us understand how macroeconomic variables influence the extreme loss risk of financial institutions. It provides an additional tool for the regulator to locate the origin of the risk.

Our work provides several contributions to the literature. Firstly, by integrating Value at Risk (VaR) with Extreme Value Theory (EVT), our systemic risk indicator presents a novel and comprehensive approach for measuring risk across the entire financial system, rather than focusing solely on individual institutions. This global perspective is crucial for understanding systemic vulnerabilities that single-entity analyses might overlook. Secondly, our method's ability to capture both systemic and non-systemic risks provides a more nuanced understanding of market dynamics, as evidenced by its successful identification of major financial crises and events that traditional models like the VIX and market VaR failed to detect comprehensively. Thirdly, the application of our indicator in analyzing the relationship between extreme losses and macroeconomic variables offers a groundbreaking tool for policymakers and financial institutions. It not only enhances the understanding of how economic policies and conditions impact financial stability but also aids in proactive risk management and policy formulation.

The paper proceeds as follows, section 2 is the literature review of the existing systemic risk measures and models, section 3 presents our data and methodology used in the paper, section 4 shows the empirical results of our indicator, section 5 is the robustness analysis using GPD, section 6 concludes the paper. An Appendix is shown at the end containing the detailed model calculation, figures, and tables that are not shown in the main text.

2 Literature review

After the 2008 financial crisis, researchers paid more attention to systemic risk, the challenge of accurately modeling and predicting extreme financial events has led to the development and evolution of various quantitative methods. Many researchers tend to meet this goal by developing systemic risk indicators, they are critical for regulators, policymakers and financial institutions in monitoring and mitigating systemic risks, for instance, Patro, Qi, and Sun (2013) make a point that daily stock return correlation is a simple, robust, forward-looking, and timely systemic risk indicator, and extreme dependence, and co-dependence can also reflect the downturns in the US financial industry (Balla, Ergen, and Migueis, 2014), the Composite Indicators of Systemic Stress (CISS) that has been developed for the Euro Area with a focus on systemic risk, which is considered to have the ability to measure systemic risk, focusing on the systemic dimension of financial stress (Altınkeski, Cevik, Dibooglu, and Kutan, 2022). These systemic risk indicators have limited predictive power and do not emphasize the significance of extreme loss, the aftermath of extreme losses can be prolonged, leading to long-term economic downturns, changes in market structures, and shifts in regulatory policies. Hence we propose a new systemic risk indicator by combining the Value-at-Risk (VaR) and Extreme Value Theory (EVT) to model the extreme events in the financial market.

Benoit, Colliard, Hurlin, and Pérignon (2017) provide a comprehensive survey on systemic risk, highlighting the central role of methodologies like VaR in the assessment of systemic risks. Duffie and Pan (1997) emphasize the ability of VaR to condense complex risk exposures into a single, understandable metric. Manganelli and Engle (2001) further explores various VaR models in finance, offering practical insights for real-world applications and stressing the importance of model selection and calibration for accurate risk assessment. However, VaR is limited to providing information about the tail of the risk distribution and cannot offer insights into other parts of the distribution. This means that VaR might fail to capture the risk of extreme events. This implies that VaR needs to be combined with other methods to offer a more comprehensive measurement and monitoring of market risk (Linsmeier and Pearson, 2000). The limitations of VaR are also critically analyzed by Danıelsson (2002), who points out the model's inadequacies in capturing extreme market conditions. Many researchers have attempted to address this issue. For instance, Embrechts, Puccetti, and Rüschendorf (2013) investigates the complexities of model uncertainty and VaR aggregation, tackling the challenges in accurately assessing risks across diverse portfolios. Adrian and Brunnermeier (2016) propose CoVaR, which focuses on the contribution of each institution to overall system risk, but CoVaR may provide a realistic approximation for smaller financial institutions and cannot capture the heteroscedasticity characteristic of financial assets, which may underestimate systemic risk (López-Espinosa, Moreno, Rubia, and Valderrama, 2015).

EVT has increasingly become a pivotal tool in financial risk management, particularly for modeling and predicting rare, extreme market events. Its application has been instrumental in understanding and preparing for market behaviors that lie outside the realm of normal fluctuations. The application of the EVT in risk management has been explored by many (Abad, Benito, and López, 2014; Carvalhal and Mendes, 2003; Bekiros and Georgoutsos, 2005, among others), it is underscored by the effective modeling the tails of distribution, which is crucial for accurate risk assessment in financial markets (Diebold, Schuermann, and Stroughair, 2000; Longin, 2005; Gilli and Këllezi, 2006; Allen, Singh, and Powell, 2011). There are some researchers who tend to combine EVT and VaR. However, most have focused on using

EVT to compute VaR. Rocco (2014) provides a comprehensive survey of the use of EVT in finance. He makes a comparison on using EVT to compute VaR and ES, concentrating on extreme quantiles of distribution rather than the distribution itself but it is crucial to model the distribution tails properly in order to predict the frequency and magnitude of extreme stock price returns (Furió and Climent, 2013; Aslanertik, Erdem, and Kurt Gümüş, 2017). Hence, we propose a method using EVT to model the VaR exceptions cross-sectional sum, Pérignon and Smith (2010) discover that VaR exceptions can accurately reflect the risk level of commercial bank transactions. Furthermore, it also forms an essential component in the metrics evaluating the disclosure quality of VaR in commercial banks. The rise of VaR Co-Exceptions could be seen as systemic co-jumps, Caporin, Kolokolov, and Renò (2017) find that the simultaneous occurrence of jumps in several stocks, often triggered by major financial news, can be indicative of short-term stock returns, correlate with sudden spikes in the variance risk premium, and lead to persistent changes in stock variances and correlations depending on the nature of the news. This perspective underscores the nuances that traditional univariate jump statistics applied to stock indices might not capture.

The continuous evolution of methodologies like VaR and EVT highlights an ongoing quest for more accurate risk modeling, especially in the face of complex market dynamics. While these methods have significantly advanced our understanding, they also underscore the need for broader perspectives that encompass systemic risks and their interplay with economic variables. Recent developments in systemic risk measures, such as SRISK proposed by Brownlees and Engle (2016), mark a pivotal advancement in quantifying potential capital shortfalls during severe market downturns. These measures, alongside studies like those by Giglio, Kelly, and Pruitt (2016), emphasize the necessity of integrating economic perspectives into financial risk analysis. The recent insights into financial uncertainty and its impact on the real economy, as explored by Dew-Becker and Giglio (2023), further contribute to this expanded view. Understanding the nuances of variance risk premium and skewness provides a more comprehensive perspective on the dynamics between financial markets and economic cycles, which is crucial for effective risk modeling and management strategies.

Our research fills the gaps in existing literature by adopting a comprehensive approach. We emphasize the VaR exception matrix, capturing the fluctuation of VaR exceptions and providing a richer context for risk modeling. Moreover, our combined application of EVT models and cross-sectional concomitant VaR exceptions (Co-Exceptions) time series offers a more thorough and robust understanding of extreme financial events. The use of knowledge graphs in analyzing systemic risk by Chen and Zhang (2023) further enriches our methodology, offering a novel perspective on the interconnectedness among financial firms and their exposure to systemic risk. By juxtaposing the findings from the aforementioned models, we ensure that our predictions are not merely a result of model-specific characteristics, but are valid across different methodologies. Thus, our study not only emphasizes the importance and limitations of traditional models such as VaR and EVT, but also demonstrates how combining systemic risk measures, financial uncertainty analysis, and innovative risk assessment methods can provide a more comprehensive risk management framework. This holistic methodology not only aids in better understanding extreme events in financial markets but also offers new insights into assessing their potential impact on the overall economy.

3 Data and Methodology

3.1 Data

In order not to work exclusively with financial institutions that survived the 2008 financial crisis and thus limit the impact of survivorship bias, we use the data on a panel of large U.S. financial firms, the same companies as Brownlees and Engle (2016). It consists of 95 financial institutions that can be classified into 4 groups: Depositories, Insurance, Broker-dealers, and Others (e.g. BlackRock). The data are observed from January 1, 2000 to December 30, 2022, so our sample contains the 2008 financial crisis, the COVID-19 health crisis, and to a lesser extent the 2011-2012 European sovereign debt crisis.

3.2 VaR with GARCH model

The first contribution of this paper is to compute a VaR exception matrix, In order to do that, the first step is to calculate the VaR of assets, VaR can estimate of how much a certain portfolio can lose within a given time period, for a given confidence level (Engle and Manganelli, 2004), The general form of VaR is as follow:

$$\mathrm{VaR}_t(\alpha) = \sigma\Phi^{-1}(\alpha) \tag{1}$$

Where $\Phi(.)$ is the distribution function of the law $\sim \mathcal{N}(0,1)$

In order to calculate the VaR, we first need to calculate the volatility of each asset at time t. There are different models that could calculate this volatility, the one we proposed is the GARCH model, which allows us to model and predict the conditional variance of the profit and loss distribution, which will then allow deriving a model or a prediction of the VaR, GARCH alone ignores the extreme tail risks, leading to underestimating systemic risk (Girardi and Tolga Ergün, 2013), but the combination of VaR with GARCH allows for possible changes over time in the linkage between individual markets and the global economy, it is more robust in assessing systemic risk (Ellis, Sharma, and Brzeszczyński, 2022). The returns are well fitted by the GARCH model partly due to the volatility clustering and GARCH does not assume that the returns are independent which allows modeling the leptokurtic property of returns. The GARCH equation is as follows:

$$\sigma^2_{i,t} = \omega + \alpha r^2_{i,t-1} + \beta\sigma^2_{i,t-1} \tag{2}$$

However, instead of working with VaR realizations computed over the estimation period of the model, we compute one-period VaR forecasts using the estimated coefficients. We thus have VaR forecasts in t knowing the set of information available in $t-1$. The daily forecast of the conditional volatility is equal to

$$\sigma^2_{i,t|t-1} = \widehat{\omega} + \hat{\alpha} r^2_{i,t-1} + \hat{\beta}\sigma^2_{i,t-1} \tag{3}$$

In order to limit the computation time, we first report the results obtained by using the whole period to estimate the models. Several GARCH models are tested during this analysis, we select the model that tested the smallest Schwarz information criterion (BIC) but also we bring to obtain non-correlated residuals ($z_{i,t}$). The first criterion leads us to select parsimonious models since the penalty term of this criterion increases more strongly (compared to the usual

Akaike criterion) when we add one more parameter to be estimated:

$$BIC = \ln\left(\frac{\sum_{i=1}^{T}\hat{\varepsilon}_{it}^{2}}{T}\right) + k\frac{\ln(T)}{T} \tag{4}$$

where k is the number of parameters of the model. The penalty term is the second element of the equation. This leads us in most cases to use the GJR-GARCH(1,1,1) model, allowing us to take into account the leverage effect, which we combine with innovations following a Student's t-t distribution with n degrees of freedom. The leverage effect could take into account the additional variance generated by negative returns compared to that generated by positive returns of the same magnitude. Also, GARCH is not able to asymmetric response of the shocks. To remedy this issue, GJR-GARCH was proposed by Glosten, Jagannathan, and Runkle (1993). The GJR-GARCH $(1,1,1)$ is defined by :

$$\sigma_{i,t}^2 = \omega + \alpha r_{i,t-1}^2 + \gamma I_{r_{i,t-1<0}}\varepsilon_{i,t-1}^2 + \beta\sigma_{i,t-1}^2 \tag{5}$$

where $I_{r_{i,t-1<0}}$ is an indicator variable such that :

$$I_{r_{i,t-1<0}} = \begin{cases} 1 & \text{if} \quad r_{i,t+1} \leq 0 \\ 0 & \text{if} \quad r_{i,t+1} > 0 \end{cases}$$

γ controls for the asymmetry of the shocks and if $\gamma = 0$, then the response to the past shock is the same; If $\gamma > 0$, then the response to the past negative shock is stronger than that of a positive one; If $\gamma < 0$, then the response to the past positive shock is stronger than that of a negative one. Finally we have the VaR calculated with GJR-GARCH model:

$$\text{VaR}_{i,t|t-1}^{\alpha} = \sigma_{i,t|t-1}^2\Phi^{-1}(\alpha) \tag{6}$$

Using a Student distribution to model the conditional return allows us to capture the excess kurtosis remaining in the latter. Indeed, in spite of the presence of a GARCH model, very often the distribution of $z_{i,t}$ remains leptokurtic, it is thus necessary to model this greater thickness in the tails of the distribution. The Student distribution allows this when its degree of freedom is close to 2, conversely when n tends towards infinity (in general $n > 30$) then this distribution tends towards the normal distribution, it is thus not necessary to assume normality on the residuals.

3.3 Exception of VaR

After estimating the VaRs, we can use them to determine the day on which a financial institution i is under financial stress. We therefore define what are called exceptions. The Value-at-Risk exception is a situation where the loss exceeds the calculated VaR for a given portfolio or investment. When the actual loss exceeds the VaR, it means that the risks of the portfolio or investment were higher than the VaR predicted. The VaR exception is an indicator that risk is not well understood or managed (Xiong, 2018). It is defined at each period by the variable $I_{i,t}$:

$$I_{i,t} = \begin{cases} 1 & \text{if} \quad r_{i,t} \leq VaR^{\alpha}_{i,t|t-1} \\ 0 & \text{if} \quad r_{i,t} > VaR^{\alpha}_{i,t|t-1} \end{cases} \tag{7}$$

where $r_{i,t}$ is the observed daily return for financial institution i, while $VaR^{\alpha}_{i,t|t-1}$ is the forecast in t knowing $t-1$ of the Value-at-Risk of financial institution i for a risk threshold α.

The exception matrix is in fact a matrix in which we calculate the VaR exception for all the assets in the portfolio (in columns) and for each day (in rows). From these VaR exceptions, we can now proceed to the computation of the cross-sectional sum. A cross-sectional sum is a vector of dimension $n \times 1$., where n is the number of dates at which we computed the exceptions. The components of this vector are computed daily on a portfolio by summing the value of the Value-at-Risk exception of each asset in the portfolio. For example, for a portfolio composed of 5 assets, the exception matrix between 01/01/2000 and 04/01/2000 is in Table 1, This exception vector is therefore a quantitative means that allows financial institutions that use it to validate or not an internal forecasting model. The exception matrix and the Co-Exceptions represent relevant information on the systemic financial risk that may threaten the market, which refers to a particular event that will cause a chain reaction with significant negative effects on the entire system, potentially leading to a general crisis in its functioning. By its magnitude, systemic risk is sufficient to cause the collapse of almost an entire financial

Table 1: Example of a VaR Exception Matrix

Time	Asset $_1$	Asset $_2$	Asset $_3$	Asset $_4$	Asset f_5	$\Sigma_{\text{Exception}}$
01/01/2000	1	0	0	1	0	2
02/01/2000	1	1	0	1	0	3
03/01/2000	0	1	1	0	0	2
04/01/2000	1	1	1	1	1	5

Notes: Columns represent the assets and the rows represent the time. The cross-sectional concomitant VaR exceptions (Co-Exceptions) of this portfolio between 01/01/2000 and 04/01/2000 is therefore the vector: $\Sigma_{Exception} = (2, 3, 2, 5)$

or economic system. The study of a daily cross-section tells us a lot about the performance of the market. The systemic risk then manifests itself in a large number of exceptions to Value-at-Risk.

3.4 EVT theory

In this section, we will model the series of the Co-Exceptions $\Sigma_{Exception}$ at a weekly level. Extreme value theory allows us to calculate the probability of occurrence of a rare phenomenon and to calculate the probability of observing a given number of simultaneous exceptions for a fixed time horizon (A. Dicks and de Wet, 2014).

The most common example used to explain the theory of extreme values is the flooding of the Seine. The level of the river has been recorded daily for several centuries. The classical approach of generalized extreme values distribution (GEV) is to build a new series of data that is aggregated by seeking the maximum value for a given block (Block Maxima Models). Thus, it is necessary to recover the highest level of the Seine by year. An alternative approach would be to consider only the daily levels of the Seine above a certain threshold and to model this series with a generalized Pareto distribution (Peaks over Threshold Models). Once the allocations are estimated, it is possible to calculate the return level of an event for a given number of blocks. In this part, we focus on the first approach and we will discuss the second approach in the robustness analysis of the next section.

As explained in the previous example, it is not possible to estimate directly the series of the Co-Exceptions. This is why we construct a series containing only maxima by finding the maximum value of the Co-Exceptions for each week of our sample. The GEV family could be combined by the Gumbel, Frechet and Weibull families :

$$H(x; \mu, \sigma, \xi) = \exp\left[-\left(1 + \xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] \tag{8}$$

defined on

$$\left\{x : 1 + \xi\frac{x-\mu}{\sigma} > 0\right\}$$

where $-\infty < \mu < +\infty, \sigma > 0$ and $-\infty < \xi < +\infty$. The location parameter μ represents the shift or offset of the distribution, determining the horizontal position of the distribution curve. It determines the location of the distribution function along the horizontal axis; The scale parameter σ controls the degree and magnitude of variation in the distribution, determining the vertical stretching or compression of the distribution curve. It determines the expansion or contraction of the distribution function; The shape parameter ξ is used to describe the shape characteristics of the distribution. It determines the thickness or density of the tails of the distribution. The shape parameter can take any real value, and different values correspond to different distribution shapes or forms.

This series thus has good characteristics to be explained by one of the three large domains of attraction of the extreme value theory. These three main domains are classified according to the value of the shape parameter:

If ξ <0, then the density belongs to the Weibull maximum attraction domain (in red in Figure 1);

If ξ =0, then the density belongs to the Gumbel maximum attraction domain (in black on Figure 1);

If ξ >0, then the density belongs to the Fréchet maximum attraction domain (blue in Figure 1), these distributions are capable of capturing thick distribution tails.

The theorem gives us the distribution of our maximum series, next we estimate the parameters using the Maximum likelihood method, there are several methods available to estimate the parameters of the GEV distribution, including maximum likelihood estimation, moment estimation, and Bayesian estimation, among others. Each estimation method has its own advantages and limitations. However, we consider maximum likelihood estimation to be a preferable approach, details of the MLE for GEV distribution see Appendix.

3.5 Return level

The return level is one of the key indicators to assess the risk of financial markets. It tells us the average frequency of extreme events above or equal to a certain threshold over a certain time horizon. By calculating the return level, we can understand the probability of extreme events occurring at a given level of risk, which helps us understand the frequency and magnitude of similar events in the past and informs future decisions.

In order to calculate the return level, we need to define the return period, the return period is the average duration during which an event of the same intensity recurs. The return period T of an event s is defined by :

$$T(s) = \frac{1}{P(X > s)} = \frac{1}{1 - F(s)} \tag{9}$$

where $P(X > s)$ denotes the probability of exceeding the s level. For example, we suppose event X has 200 observations over a period of 40 years. The probability that the return period of this event is 100 years verifies:

$$\begin{cases} P(X \geq s) = \dfrac{x}{200} \\ 100 = \dfrac{40}{x} \end{cases}$$

Finally, we obtain

$$P(X \geq s) = \frac{40}{200} \times \frac{1}{100} = 0.002$$

It is simpler to say that we are interested in an event that is observed every 100 years rather than an event whose probability is 0.002.

The return level L_t exceeded once every T periods verifies :

$$P(X \geq \mathrm{L}_t) = \frac{1}{T} \tag{10}$$

We thus obtain:

$$L_t = \bar{F}^{-1}\left(\frac{1}{T}\right) \tag{11}$$

where $\bar{F}(x) = 1 - F(x)$. Finally, we have the return level (See detailed calculation in

Appendix):

$$L_t = \mu - \frac{\sigma}{\xi} \times \left(1 + \log\left(1 - \frac{1}{T}\right)^{-\xi}\right) \tag{12}$$

The calculation of the return level provides a benchmark that allows us to assess whether current market conditions have deviated from historical levels and to evaluate possible future risks based on past experience. Furthermore, it can also be used to verify the fitness of the generalized extreme value distribution model. By comparing with observed data, we can evaluate the performance of the fitted model at different return levels and check the fitness and accuracy of the model.

4 Empirical results

In this section, we first report the daily VaR Exceptions of the 95 financial institutions from 2000 to 2022, and then the modeling results of the GEV theory.

4.1 Daily VaR Exceptions

Figure 2 is the Co-Exceptions in a daily basis, most of the exceptions were concentrated during the 2007-2009 financial crisis, the 2010 European sovereign debt crisis, and the COVID-19 crisis. The subprime crisis began to emerge in February 2007 when HSBC increased its bad debt provisions by 1.8 billion dollars for its subprime mortgage business in the United States, reaching the first peak of VaR Exceptions. Since then, several financial institutions such as Bear Stearns, BNP Paribas, and Deutsche Bank suffered losses due to the subprime crisis. Companies like New Century Financial Corporation, IndyMac Bank, and Lehman Brothers filed for bankruptcy, while others were nationalized. In Figure 2, the Co-Exceptions only slightly decreased by the end of 2009, representing the end of the 2008 financial crisis. From the graph, it can be seen that around May 2010, Co-Exceptions increased again, marking the beginning of the European debt crisis, several banks, including the four major Greek banks, Barclays Bank, and HSBC, had their ratings downgraded. In 2016, major banks like Deutsche Bank and Banca MPS of Italy incurred significant losses, explaining the occurrence of a new peak in VaR Exceptions. Due to the COVID-19 crisis in 2020, the economy experienced a downturn and turbulence, explaining the peak of the third round of VaR Exceptions.

Table 2 represents the top 20 daily Co-Exceptions with their financial events. While Co-Exceptions are typically employed as indicators of systemic risk, the table reveals that these exceptions are not solely attributable to the intrinsic structure of financial markets or the frailties within the financial system. On the contrary, a multitude of these exceptions can be traced to specific exogenous events, ranging from political decisions such as Brexit, to sudden economic disturbances like oil price skirmishes, global health crises exemplified by the COVID-19 pandemic, and even including natural disasters and conflicts. VaR exceptions capture not only systemic risk but also non-systemic risks that are intimately tied to market operations. Events such as oil price wars or credit rating downgrades may deliver immediate shocks to the markets, yet they do not invariably pose a long-term threat to the entire financial system. Additionally, this underscores the financial market's susceptibility to non-financial occurrences, like political resolutions or public health emergencies, which can precipitate severe market fluctuations without forewarning.

4.2 GEV model results

Figure 3 represents the density of the weekly maxima of VaR Exceptions. From the graph, it can be observed that for the majority of the observation period, the financial system is relatively stable, with weekly maxima ranging from 0 to 20 and the density decreasing with higher weekly maxima. The portion where weekly maxima exceeds 40 represents significant fluctuations in nearly half of the financial institutions during the observation period, which is an area that we should pay attention to.

We then calculate GEV parameters using the maximum likelihood estimation method. The GEV model has three parameters: the location parameter, scale parameter, and shape parameter. When the shape parameter ξ is zero, the GEV model degenerates into the Gumbel distribution. The Gumbel distribution has a unimodal, right-skewed shape and is suitable for describing extreme values with a longer right tail. When ξ is greater than zero, the GEV model corresponds to the Fréchet distribution. The Fréchet distribution has a right-skewed shape with a heavier tail and is suitable for describing extreme values with a larger tail risk. When ξ is less than zero, the GEV model corresponds to the Weibull distribution. The Weibull distribution has a left-skewed shape with a heavier tail and is suitable for describing extreme

Table 2: Top 20 daily VaR Co-Exceptions

Date	VaR Exceptions	Event
2007/2/27	65	Shanghai Stock Exchange drop 9%, global impact
2007/3/13	51	Housing slump affects the financial sector, market volatility
2007/7/24	50	Housing market sags, subprime crisis intensifies
2007/8/3	55	The subprime mortgage issue worsened further
2007/11/7	50	General crash in global stock markets
2008/9/29	56	Market crash after bailout bill failure
2010/8/11	53	The European sovereign debt crisis
2011/8/4	59	Eve of U.S. credit rating downgrade by S&P
2011/8/8	67	U.S. and global markets fall post U.S downgrade
2015/6/29	54	Greek debt crisis peak impacts European banks
2015/8/24	63	Flash crash
2016/6/24	61	Brexit vote causes market turbulence
2018/2/5	59	2018 stock market correction begins
2018/2/8	58	Continued market adjustment
2018/3/22	53	Trade war fears due to U.S.-China tensions
2018/12/4	55	The stock market declined
2019/8/14	52	Yield curve inversion signals potential recession
2020/2/24	52	COVID-19 pandemic starts, economic concerns
2020/3/9	60	Oil price war and COVID-19 impacts markets
2020/6/11	58	Market drops due to COVID-19 case increases

Notes: This table represents the top 20 daily VaR Co-Exceptions.

values with a smaller tail risk. After fitting the weekly maxima to the GEV model using the maximum likelihood estimation, the following parameter results were obtained in Table 3.

The shape parameter is greater than 0, indicating that it belongs to the Fréchet distribution, which can describe extreme values with a larger tail risk. This aligns with our hypothesis.

The GEV diagnostic pictures are presented in Figure 4, Quantile-Quantile (Q-Q) Plot (top left) compares the quantiles of the empirical data against the quantiles of the theoretical GEV distribution, it appears that for the most part, the data conform to the GEV model quite well except for the tail, which seems to be heavier than the model predicts, indicating potential underestimation of extreme values. The probability plot (top right) is similar to the Q-Q plot

Table 3: GEV parameters

Parameters	Location	Scale	Shape
Value	3.099	3.779	0.878

Notes: The 3 GEV parameters are calculated using whole sample period, shape parameter is greater than 0 meaning the model follows a Fréchet distribution.

and shows some deviation at the higher quantiles. In the density plot (bottom left), the density of the empirical data (solid line) is very close to that of the fitted model (dashed line), but in the return level plot (bottom right), the empirical points at the extreme end significantly deviate from the model, suggesting that the model may not fully capture the most extreme events.

In this context, we propose to use a rolling window to estimate the model and calculate the return level. We take into consideration the dynamics of the data, as financial markets data often exhibit dynamics and time-varying behavior, capturing market behavior and volatility across different time periods. By using the rolling window approach, we can apply calculations to subsets of data from different time periods, better reflecting the dynamic nature of the data. Financial markets operate under different market conditions, also it allows us to calculate the recurrence levels for each window period, capturing changes in different market environments and gaining a more comprehensive understanding of market changes and risks.

We select a rolling window size of 2 years because it covers a longer time span, allowing for better capturing of long-term trends and cyclical changes. Compared to shorter-term rolling windows, our approach provides more comprehensive data information, as shorter-term windows may result in smaller sample sizes and be more susceptible to data noise and outliers. Furthermore, it is better to resist such disturbances and provide more reliable estimation results. Additionally, it can reduce estimation errors caused by data volatility within a single time period. By taking the average of multiple time periods, more stable parameter estimation results can be obtained, reducing estimation biases caused by randomness.

Figure 5 represents the parameters of the GEV model using a 2-year rolling window. It shows that regardless of the observation period, the shape parameter is greater than 0, indicating that it follows the Fréchet distribution. The mean values of the three parameters are shown in Table 4.

Table 4: GEV parameters using rolling window

Parameters	Location	Scale	Shape
Mean Value	3.446	4.040	0.822

Notes: The 3 GEV parameters are calculated using 2-year rolling window, the mean of the shape parameter is greater than 0, indicating that the model always follows a Fréchet distribution in the different time periods.

The location parameter and scale parameter are larger than those calculated without using the rolling window, indicating that the rolling window better captures the overall risk level and volatility. A larger scale parameter value suggests higher volatility and risk. However, the shape parameter calculated through the rolling window is slightly smaller, indicating a lower tail risk and a lower probability of extreme events. We consider the parameter results obtained using the rolling window method to be more reliable because the rolling window method captures the effects of time changes. Financial markets and economic conditions may change over time, leading to variations in the probability of extreme events and tail risk. Therefore, using the rolling window method can better reflect the impact of time changes on the shape parameter.

Figure 6 illustrates how the estimated risk level, as quantified by the return level of VaR Co-Exceptions, changes over time. The return level reflects the estimated maximum loss for a given return period, in this context, relates to six weeks. It is evident from the graph that the return level is not static but fluctuates, indicating that the market's perception of risk is dynamic and influenced by numerous factors. The ebb and flow of the return levels might be linked to changes in market conditions, such as variations in volatility, macroeconomic announcements, or significant geopolitical events. The peaks in the graph generally correspond to periods of market stress or financial instability. During such times, market participants anticipate higher potential losses, which is reflected in increased return levels. For example, the sharp rise around the 2007-2009 period likely corresponds to the global financial crisis, where market volatility and risk aversion reached extreme levels. Conversely, the troughs may reflect periods of relative market calm or confidence, where lower return levels suggest a lower expected frequency and severity of VaR Co-Exceptions. The trajectory of the return level over time can provide insights into the evolving nature of market risk. An overall downward trend might suggest that markets are becoming more efficient at pricing risk or that risk mitigation mechanisms are becoming more effective. Alternatively, it could signal a period of increasing market stability or decreasing volatility. It's also noteworthy that the reaction of the return levels to recent events (like the COVID-19 pandemic), shows that even in an era of advanced financial instruments and risk management strategies, markets remain vulnerable to unexpected, high-impact events.

We compared the return level with the VIX index in Figure 7, The VIX index is commonly viewed as a measure of market panic, with higher values indicating greater expected risk and uncertainty in the market. During certain periods, the return levels are closely correlated with peaks in the VIX. This suggests that expectations of VaR Co-Exceptions increase with rising market pressures. This correlation might indicate that the return level, as a measure of risk, is capable of capturing changes in market volatility and investor sentiment. Peaks are usually associated with extreme events in the financial markets, such as the global financial crisis of 2008, the European debt crisis of 2010, and the COVID-19 pandemic in 2020. During these periods, the VIX index rose significantly, with return levels showing a similar upward trend, reflecting extreme uncertainty and risk aversion in the market during these times. The return levels of VaR Co-Exceptions show a smoother and more sustained trend, while the VIX displays more intense and brief peaks. This is because the VIX reflects immediate market sentiment, whereas return levels are statistical measures based on historical data, containing more information and being more robust to short-term noise.

Figure 8 displays a comparison between the GEV return level calculated using a 6-week return period and the SP 500 market VaR. It is observed that while there are significant spikes in the GEV return level, corresponding large spikes do not appear in the market VaR. This phenomenon could be attributed to the inclusion of numerous smaller-scale companies when calculating VaR Co-Exceptions, whose volatility is not substantial enough to impact the market VaR significantly. Notably, there is a pronounced spike in the GEV return level chart around mid-2016, which is not present in the market VaR. This suggests that our method captures a more comprehensive picture of the risk fluctuations within the system, particularly the risks of smaller entities that might be overlooked by market VaR. Hence, our approach offers a broader perspective in identifying systemic risks, capturing hidden risk points that could pose a threat to overall market stability.

Figure 9 presents the return probability of exceeding 50 VaR Co-Exceptions. We can estimate the probability of having 50 VaR Co-Exceptions occurring in any given week. Unlike the return level, which indicates the potential maximum loss that might be reached, the return probability refers to the likelihood that a given threshold will be met or exceeded within a specific future time frame. This probability itself can fluctuate under different market conditions, and these fluctuations are typically associated with market instability and impending potential risks.

4.3 Regression

In this section, we will explain return levels (for a monthly 6-week return period) using macroeconomic and systemic variables. To determine the factors influencing our return levels, we have selected these potential variables in Table 5.

Table 5: Explanatory variables

Variables	Signification
INTDSRUM193N	Interest rate
UNRATE	Unemployment rate
INDPRO	Industrial production index
VOL	Volatility
EXUSEU	Euro-dollar exchange rate
CPIAUCNS	Consumer Price Index for All Urban Consumers
PCE	Personal Consumption Expenditures Price Index
FEDFUNDS	Effective federal funds rate
CISS	Composite indicator of systemic stress
EXPINF1YR	Expected inflation

Notes: This table explains the explanatory variables selected to estimate the return levels of a monthly 6-week return period.

Initially, we examined the correlations between our different explanatory variables to make an initial selection. Figure 11 is the correlogram of the variables, correlogram is a graph showing the correlations between variables in a data set, it is useful for identifying variables that are highly correlated and for assessing the nature of the relationship between variables. Figure 11 shows that there is a very strong correlation (0.98) between the PCE and CPIAUCNS variables. In fact, these two variables relate to consumer product prices. To avoid any inversion difficulties, we drop the PCE variable. Next, we perform a residual analysis to see how the model fits the data by examining the goodness of fit, this is to check whether the assumptions of the linear Gaussian model are satisfied. As shown in Figure 12, the residuals of the Residuals vs Fitted plot are randomly distributed around the center line without any apparent patterns or systematic structure, suggesting that the linear assumption is appropriate. The Normal Q-Q plot indicates that most points do indeed follow a straight line, but there is a slight deviation at the ends, especially on the right tail, which may suggest that the tail of the residual distribution is heavier than that of a normal distribution. In the Scale-Location plot, we observe that the variation in residuals does not appear to significantly increase or decrease as the fitted values increase, which is a positive sign. The Residuals vs Leverage plot is used to detect influential observations that may disproportionately influence the regression estimates; in the plot, only one point (labeled as 124) has both high leverage and Cook's distance. In summary, these diagnostic plots show that the model is well-fitted.

The results of our regression are illustrated in Table 6, The Consumer Price Index (CPIAUCNS), the Federal Funds Rate (FEDFUNDS), and the Interest Rate (INTDSRUSM193N) all have inverse relationships with return level. This suggests a higher inflation is associated with lower risk levels or a reduction in the severity of VaR Co-Exceptions. This phenomenon could stem from the impact of inflation on the real value of financial assets or the anticipated contractionary monetary policies. A higher interest rate set by central bank policies is associated with a lower return level. This correlation may underscore the central bank's instrumental role in stabilizing the economy and managing systemic risks, as heightened interest rates typically aim to temper economic growth and inflation, potentially fostering a more stable financial milieu. Furthermore, increased market rates may compensate for reduced liquidity and credit risks.

The Composite Indicator of Systemic Stress (CISS), Industrial Production (INDPRO), and Unemployment Rate (UNRATE) show a direct correlation with the return level. An upsurge in systemic stress corresponds with elevated return levels, signifying that systemic risks are a pivotal determinant of the risk levels captured by the sum of VaR Co-Exceptions. Industrial production, an indicator of economic expansion, may be allied with higher financial risk levels. This could be attributable to cyclical dynamics where periods of economic growth could culminate in overheating and an increased probability of corrective downturns. Elevated unemployment rates bolster return levels, meaning that economic recessions characterized by higher unemployment are perceived as epochs of intensified financial risk.

In summary, macroeconomic indicators, particularly those reflecting monetary policy, inflation, and systemic stress, are paramount in determining the financial system's risk level as captured by the sum of VaR Co-Exceptions. These findings emphasize the intricate interplay between the macroeconomic milieu and systemic risks as gauged by VaR Co-Exceptions. The research reveals that the central bank's monetary policy, as reflected in interest rates and the federal funds rate, along with systemic stress indicators, plays an indispensable role in the risk landscape of the financial system as perceived through the VaR Co-Exceptions. These insights are instrumental for central banks, financial regulators, and policymakers dedicated to understanding and mitigating systemic risks within the financial system.

5 Robustness analysis

In our robustness analysis, we transition our model to the Peaks Over Threshold (POT) approach. This methodology employs a threshold to segregate extreme values from the dataset, facilitating the modeling of the distribution's tail for values exceeding this threshold. According to Pickands' Theorem, for a sufficiently high threshold u, the distribution of exceptions converges to a Generalized Pareto Distribution (GPD) characterized by specific shape and scale parameters. The GPD family, parameterized by $\xi \in R$ and $\sigma_u > 0$ is defined by:

$$G_{\xi,\sigma_u}(y) = \begin{cases} 1 - \left(1 + \xi \frac{y}{\sigma_u}\right)^{\frac{-1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - exp\left(\frac{-y}{\sigma_u}\right) & \text{if } \xi = 0 \end{cases} \tag{13}$$

subject to $1 + \xi \frac{y}{\sigma_u} > 0$

Selecting an optimal threshold necessitates a balance between bias and variance. The threshold u must be sufficiently high to allow for theoretical convergence, yet low enough to retain an ample number of exceedances for robust estimation. The mean excess function for a random variable X over a threshold u is defined by:

$$e(u) = E(X - u \mid X > u) \tag{14}$$

The return level of the GPD model (See details in Appendix):

$$L_t = u + \frac{\sigma}{\xi} \times \left(\left(\frac{1}{T \times P(X > u)} \right)^{-\xi} - 1 \right) \tag{15}$$

The Peaks Over Threshold model with the Generalized Pareto Distribution offers a robust framework for estimating the return level and return probability of extreme financial events. By calibrating the model with the optimal threshold and employing maximum likelihood estimation, we refine our analysis of the tails of the loss distribution, which is crucial for comprehensive risk management in financial markets. This methodological robustness check ensures the solidity of our findings and enhances the credibility of our risk assessment approach.

The return probability using GPD model is in Figure 10, it reveals an intermittent pattern of elevated probabilities, with peaks corresponding to tumultuous market episodes. These findings corroborate the predictive efficacy of the GPD model and underscore its aptitude for apprehending the risk of rare yet impactful events. The observed fluctuations in return probabilities are consistent with the market's historical propensity for abrupt transitions, aligning with the overarching narrative of financial markets' susceptibility to sudden shifts in sentiment and economic conditions.

The GPD model, as applied in this robustness check, emerges as a vital tool for financial risk assessment, particularly in modeling the behavior of market extremes. The consistency between the GPD model outputs and our financial indicator using GEV model reinforces the resilience of our risk evaluation framework, thereby affirming the robustness of our approach in the measurement and interpretation of financial risk within the broader context of systemic stability.

6 Conclusion

In this paper, we propose a new systemic risk indicator from a global perspective by integrating Value at Risk (VaR) with Extreme Value Theory (EVT). It improves the inadequacy of traditional models for modeling extreme events, offers a comprehensive view of systemic risk. And also it captures not just the magnitude but also the probability of extreme financial losses across a broad spectrum of financial institutions. A distinctive feature of our approach is that we use predicted volatility to model the indicator, which provides a forward-looking view.

We demonstrate how the return level can serve as a robust indicator for extreme losses in the financial market. This method proves superior in capturing both systemic and non-systemic risks, we show its effectiveness of identifying major financial crises and specific market events that other indicators, like the VIX and market VaR, fail to comprehensively detect.

Additionally, our research makes a substantial contribution by establishing a clear link between extreme financial losses and key macroeconomic variables. This analysis not only sheds light on the underlying dynamics between economic policies and financial stability but also provides invaluable insights for policymakers in devising strategies to mitigate systemic risk.

In conclusion, our research contributes to the systemic risk assessment, it is importance to integrate diverse methodologies for a holistic understanding of financial markets. This comprehensive framework not only enhances our grasp of extreme events in financial markets but also paves the way for new insights into their potential impact on the overall economy.

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Table 6: **Regression results on return levels**

Notes: *p<0.1, **p<0.05, ***p<0.01; t-value in parentheses. The dependent variable of each column is the monthly return level calculated by averaging the weekly return level of 6 weeks return period.

	(1)	(2)
Variables	RL1	RL2
Constant	8.8778	14.2439
	(0.900)	(1.676)
CPIAUCNS	-0.0556*	-0.0690**
	(-1.802)	(-2.313)
FED FUNDS	-1.5623***	-1.4014***
	(-5.038)	(-5.010)
INTDSRUSM193N	-0.9015***	-0.9802***
	(-3.780)	(-4.665)
CISS	18.0763***	14.6053***
	(6.216)	(9.438)
INDPRO	3.2312***	3.2478***
	(15.044)	(15.926)
UNRATE	0.7284***	0.7305**
	(2.880)	(3.116)
EXUSEU	1.7312	
	(0.576)	
VOL	-0.0689	
	(-1.220)	
EXPINF1YR	0.6513	
	(1.214)	
Observation	170	170
$Adj.R^2$	0.865	0.864

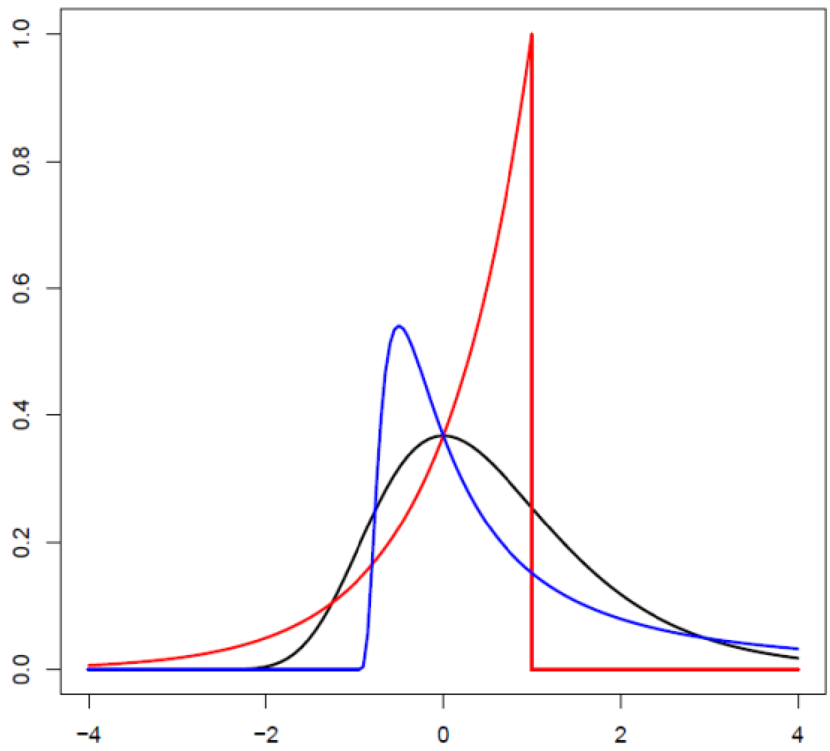

Figure 1: Example of densities associated with an extreme-value distribution

Notes: This figure shows the 3 distributions in GEV model, the red line is Weibull distribution(when shape parameter ξ <0), the black line is Gumbel distribution(when shape parameter ξ =0), the blue line is Fréchet distribution(when shape parameter ξ >0).

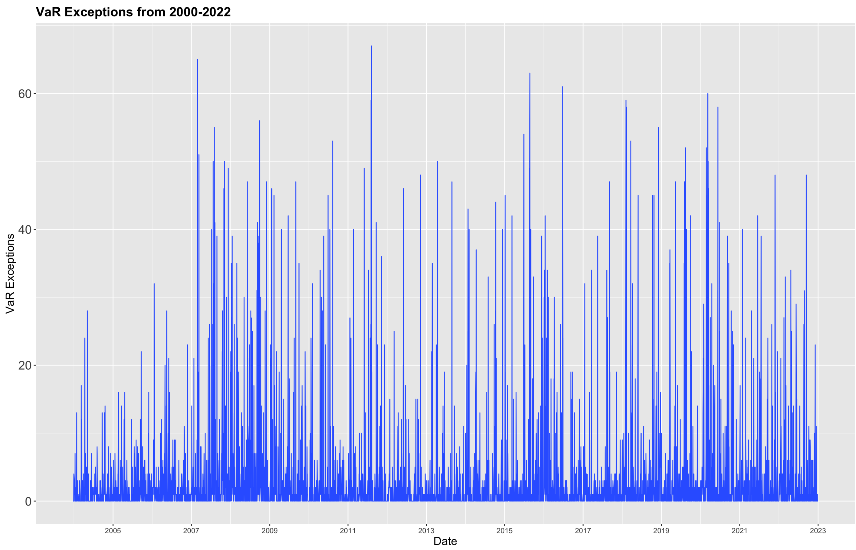

Figure 2: Daily VaR Co-Exceptions

Notes: This figure shows daily VaR Co-Exceptions calculated by a 3-year rolling window GJR-GARCH, it predicts correctly the financial events in the sample period, and the peaks in the figure indicate that the volatility of most financial institutions is higher than usual, therefore has higher risks.

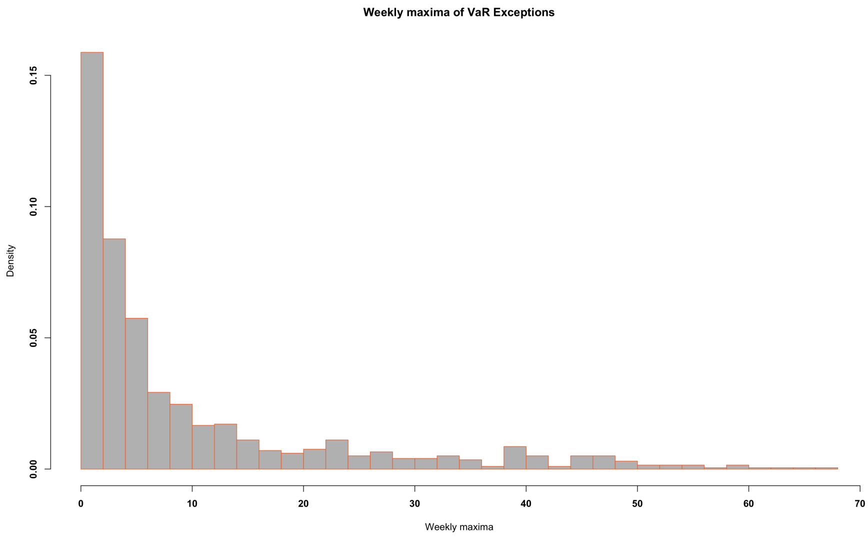

Figure 3: Density of weekly maxima

Notes: This figure represents the density of the weekly maxima of VaR Co-Exceptions. For most of the observation period, the financial system is relatively stable, with weekly maxima ranging from 0 to 20 and the density decreasing with higher weekly maxima.

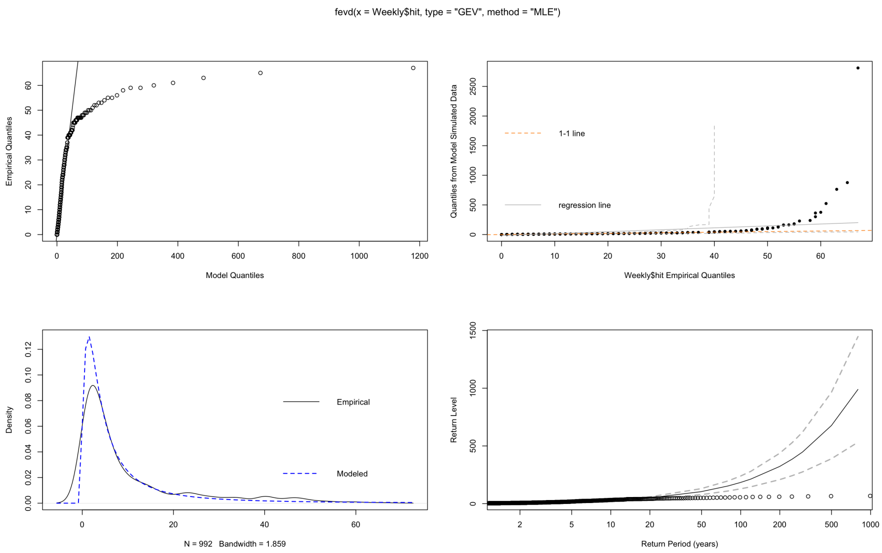

Figure 4: Diagnostic pictures of GEV model

Notes: The 4 pictures are the diagnostic graph of the GEV model, respectively the Quantile-Quantile (Q-Q) Plot (top left), the probability plot (top right), the density plot (bottom left), and the return level plot (bottom right).

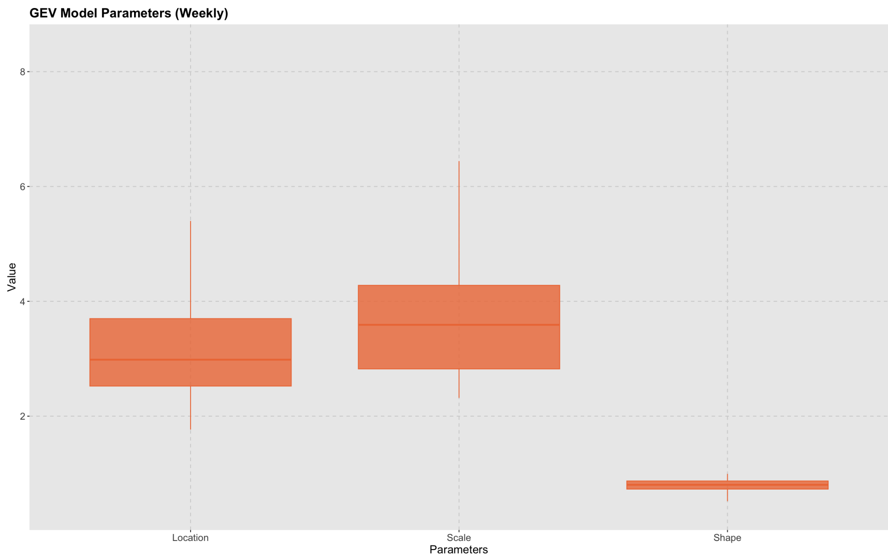

Figure 5: Parameters of GEV using rolling window

Notes: This figure shows the 3 parameters of GEV estimated by using rolling window, regardless of the observation period, the shape parameter is greater than 0, indicating that it follows the Fréchet distribution, has a right-skewed shape with a heavier tail, and is suitable for describing extreme values with a larger tail risk.

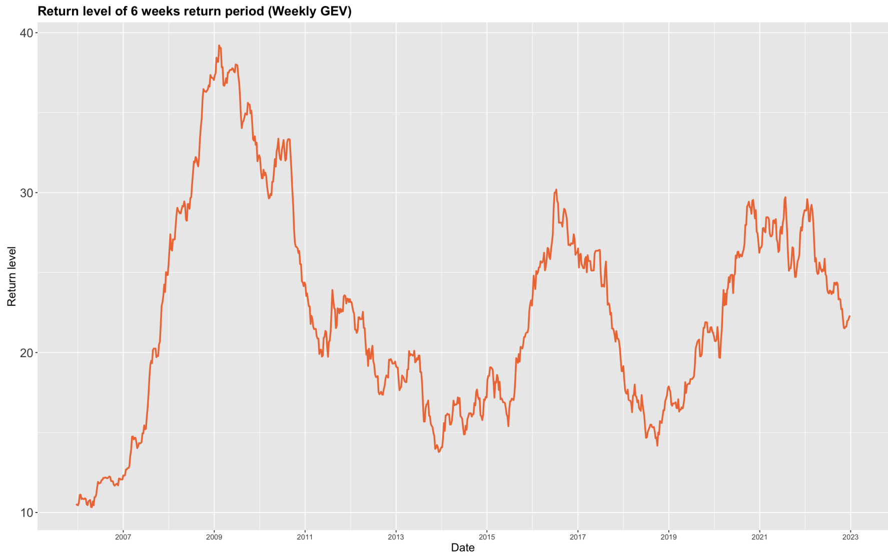

Figure 6: Return level of 6 weeks return period

Notes: This figure represents the return level for a 6-week return period. the return level effectively predicts the occurrence of major financial events. During the 2008 financial crisis, the 2010 European sovereign debt crisis, and the COVID period, the return level increased, indicating that significant financial and economic instability occurs when systemic risk events happen.

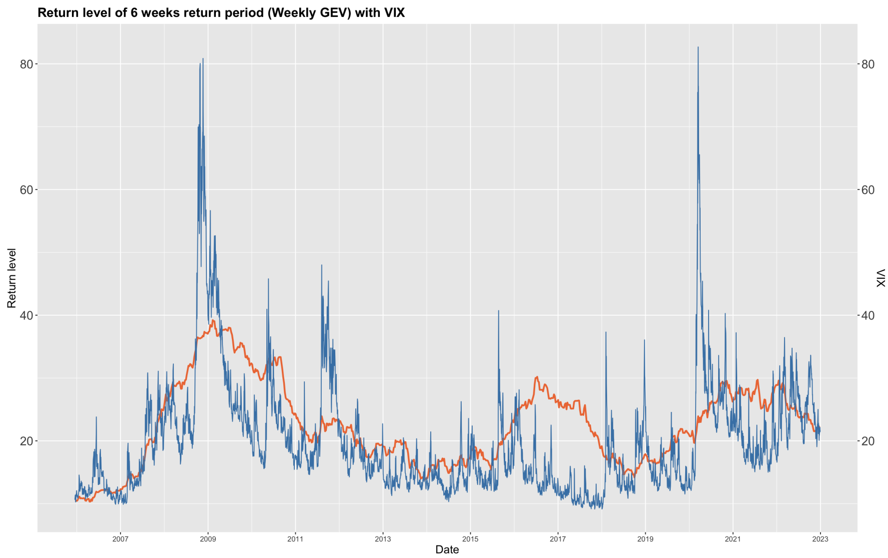

Figure 7: Return level and VIX index

Notes: This figure represents the return level for a 6-week return period compared with the VIX index. It often moves in tandem with the return levels of VaR Co-Exceptions, especially during financial crises like those in 2008, 2010, and 2020, suggesting that these levels effectively capture shifts in market volatility and sentiment.

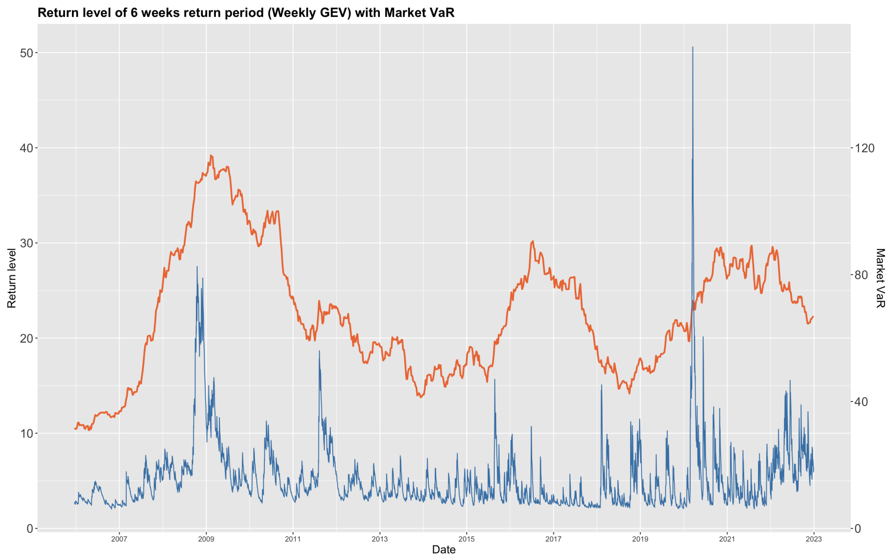

Figure 8: Return level and VaR of the Market

Notes: This figure represents the return level for a 6-week return period compared with the VaR of the market revealing pronounced spikes not mirrored in the market VaR, highlighting our method's ability to capture a broader spectrum of risks, including those from smaller entities typically overlooked in market analyses.

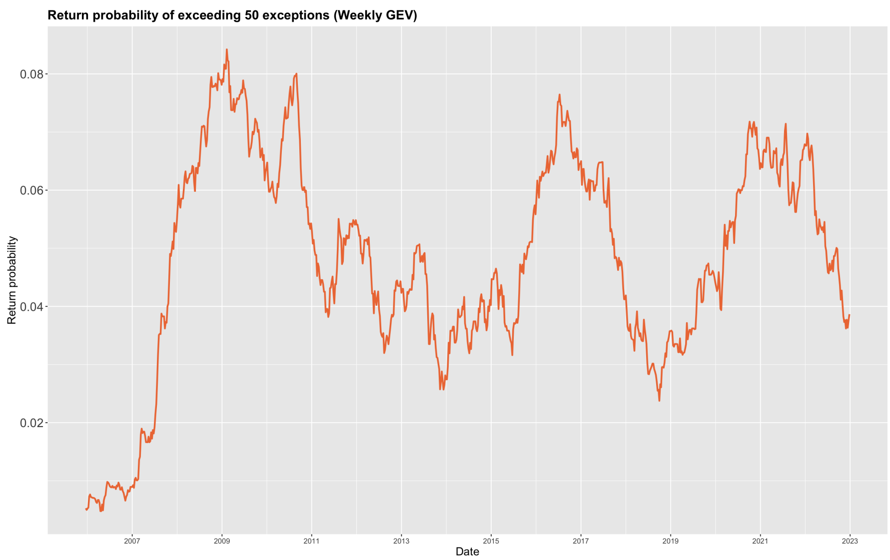

Figure 9: Return probability of 6 weeks return period

Notes: This figure represents the return probability of a 6-week return period. It refers to the likelihood that a given threshold will be met or exceeded within a specific future time frame.

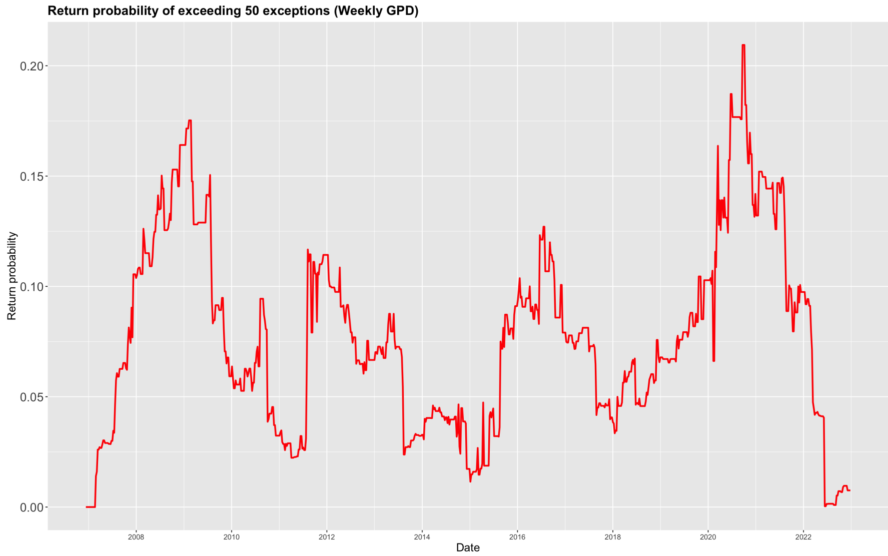

Figure 10: Return probability of 6 weeks return period

Notes: This figure represents the return probability of a 6-week return period using the GPD Model.

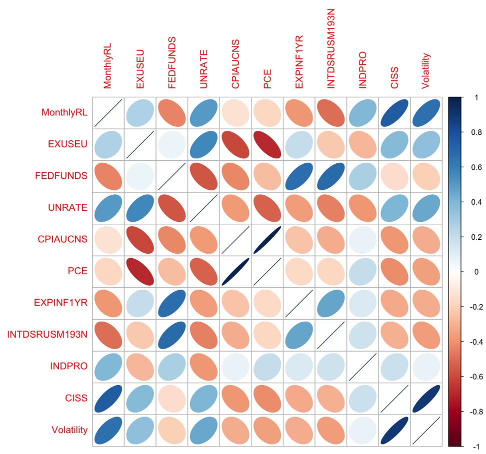

Figure 11: Correlogram of the explanatory variables

Notes: This figure represents the correlogram of the explanatory variables. there is a very strong correlation (close to 1) between the PCE and CPIAUCNS variables, these two variables relate to consumer product prices, to avoid any inversion difficulties, we will keep the CPIAUCNS variable.

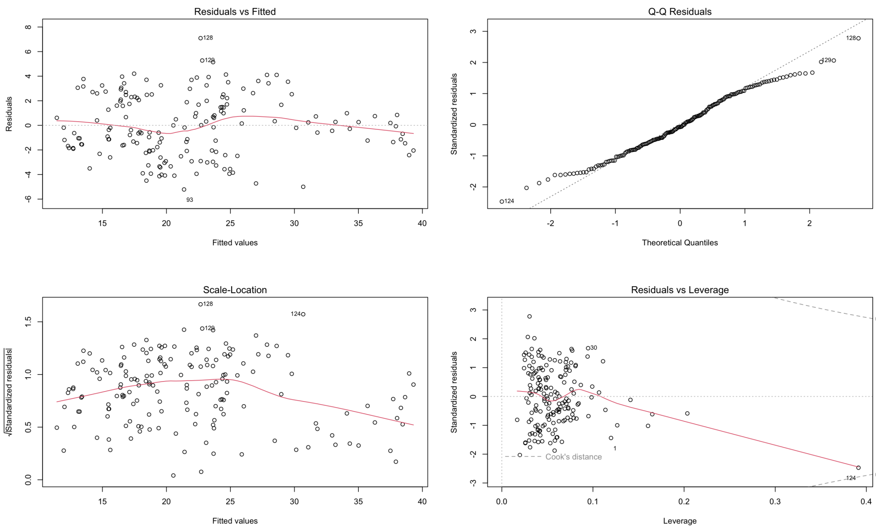

Figure 12: Residual analysis

Notes: This figure shows the residual analysis of the regression. Residuals remain globally uniformly distributed on both sides of 0. In the bottom left-hand chart, the red curve is "horizontal", with the points evenly distributed around it. In the QQ plot, the points are aligned around the first bisector, confirming that the residuals follow a normal distribution.

Appendix A Appendix

Appendix A.1 MLE for the GEV distribution

We consider maximum likelihood estimation to be better for the GEV distribution for the following three main reasons: Firstly, it is the only method that can adapt to model variations and is applicable to different approaches for modeling extremes. While different extreme modeling methods may yield different models and representations of the maximum likelihood estimates may vary, the essence of the method remains unchanged. Secondly, it allows for integrating various relevant information into statistical inference. This enables the incorporation of different types of information to improve the estimation process. Last but not least, the most critical aspect is that maximum likelihood estimation exhibits excellent large-sample properties and provides a measure of uncertainty for the estimation method, due to its flexibility, ability to incorporate diverse information, and good properties in large samples, we consider maximum likelihood estimation to be a favorable approach for estimating GEV parameters

In the case $\xi \neq 0$, the log-likelihood for a sample $X_1, \ldots, X_n$ of i.i.d. variables following a GEV is written as:

$$L(\sigma, \mu, \xi) = -n \log(\sigma) - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{n} \log\left(1 + \xi \frac{X_i - \mu}{\sigma}\right) - \sum_{i=1}^{n} \left(1 + \xi \frac{X_i - \mu}{\sigma}\right)^{\frac{-1}{\xi}} \tag{A1}$$

In condition of $1 + \xi \frac{X_i - \mu}{\sigma} > 0, i = 1, \ldots, n$

In the case $\xi = 0$, the log-likelihood for a sample $X_1, \ldots, X_n$ of i.i.d. variables following a

GEV thus becomes :

$$L(\sigma, \mu, \xi) = -n\log(\sigma) - \sum_{i=1}^{n} \exp\left(-\frac{X_i - \mu}{\sigma}\right) - \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right) \tag{A2}$$

The estimators $(\hat{\sigma}, \hat{\mu}, \hat{\xi})$ of (σ, μ, ξ) are obtained by maximizing the above 2 equations. Since the support of the density of a GEV depends directly on the value of the unknown parameters (σ, μ, ξ), the usual regularity conditions underlying the asymptotic properties of maximum likelihood estimators are not satisfied. It is therefore not possible to find an explicit expression for $(\hat{\sigma}, \hat{\mu}, \hat{\xi})$, but it is possible to obtain an approximate value of these estimators by numerical solution.

Appendix A.2 Return level calculation

We recall that the return level $Ł_t$ exceeded once every T periods verifies :

$$P\left(X \geq Ł_t\right) = \frac{1}{T} \tag{A3}$$

We thus obtain:

$$L_t = \bar{F}^{-1}\left(\frac{1}{T}\right) \tag{A4}$$

where $\bar{F}(x) = 1 - F(x)$. Consequently, the return level L_t is a quantile of order $\alpha = \frac{1}{T}$. In the Block Maxima model, we take as a distribution the family of generalized extreme values (if $\xi \neq 0$) :

$$H_{\xi,\sigma,\mu}(x) = \exp\left(-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right) \tag{A5}$$

In the case of a GEV distribution. We have:

$$P\left(X > \text{Ł}_t\right) = 1 - P\left(X \leq \text{Ł}_t\right) = 1 - \exp\left(-\left[1+\xi\left(\frac{\text{Ł}_t-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right) \tag{A6}$$

Thus, the level L_t exceeded once every T period is the solution of :

$$1 - \exp\left(-\left[1+\xi\left(\frac{L_t-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right) = \frac{1}{T}$$

We then obtain the equation of the return level:

$$L_t = \mu - \frac{\sigma}{\xi} \times \left(1 + \log\left(1 - \frac{1}{T}\right)^{-\xi}\right) \tag{A7}$$

Appendix A.3 GPD model of robustness check

When a GPD distribution function falls within one of the domains of attraction for the generalized extreme values (Fréchet, Gumbell or Weibull) then there exists a strictly positive function σ_u and a parameter $\xi \in R$ such that :

$$\lim_{u \rightarrow {}_F} \sup_{0 \leq y \leq x_F - u} \left|F_u(y) - G_{\xi,\sigma_u}(y)\right| = 0 \tag{A8}$$

Consequently, the distribution of exceptions over a large threshold u can be closely approximated by the GPD:

$$F_u \approx G_{\xi,\sigma_u} \tag{A9}$$

GPD parameters are uniquely determined by GEV parameters. The shape parameter ξ is identical to the GEV shape parameter and the scale parameter σ_u is a function of the GEV location and shape parameters:

$$\sigma_u = a + \xi(u - b) \tag{A10}$$

Selecting an optimal threshold could also be seen in the main text:

$$e(u) = E(X - u \mid X > u) \tag{A11}$$

Let Y be a random variable with a Generalized Pareto distribution function G_{ξ,σ_u}, then its mean function of excesses $e\left(u_0\right)$ beyond a threshold $u_0 < x_F$ is given by:

If this statement is true for the threshold u_0, it will be true for any other threshold $u > u_0$. Thus, for all $u > u_0$, we obtain the following equation:

$$e(u) = \frac{\sigma_u}{1-\xi} \tag{A12}$$

$$= \frac{\xi\left(u - u_0\right) + \sigma_{u_0}}{1-\xi} \tag{A13}$$

Finally, for all $u > u_0$, e(u) is a linear function in u

Upon determining the optimal threshold, we can model the exceptions that tend towards a GPD. We will estimate the parameters of the GPD using the maximum likelihood method, akin to our approach with the GEV model.

In the case $\xi = 0$, the log-likelihood for a sample $Y_1, \ldots, Y_n$ of i.i.d. variables following a GPD is written :

$$\log L(\sigma, \xi) = -n \log(\sigma) - \left(\frac{1}{\sigma}\right) \sum_{i=1}^{n} (y_i) \tag{A14}$$

In the case $\xi \neq 0$, the log-likelihood for a sample $y_1, \ldots, y_n$ of i.i.d. variables following a GPD is written :

$$\log L(\sigma, \xi) = -n \log(\xi) - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{n} \log\left(1 + \xi \frac{Y_i}{\sigma}\right) \tag{A15}$$

The subsequent step is to calculate the return level, in the POT model, we take the Generalized Pareto Distribution (If $\xi \neq 0$) :

$$G_{\xi,\sigma_u}(y) = 1 - \left(1 + \xi \frac{y}{\sigma_u}\right)^{\frac{-1}{\xi}} \tag{A16}$$

Let X be a random variable whose distribution function is a GPD. For a threshold u, we have for all $L_t > u$:

$$P\left(X > L_t \mid X > u\right) = 1 - P\left(X \leq L_t \mid X > u\right) \qquad = \left(1 + \xi \frac{L_t}{\sigma_u}\right)^{\frac{-1}{\xi}} \tag{A17}$$

Then we have:

$$P\left(X > L_t, \mid X > u\right) = \frac{P\left(X > L_t, X > u\right)}{P(X > u)} \qquad = \frac{P\left(X > L_t\right)}{P(X > u)} \tag{A18}$$

Thus, the L_t level exceeded once every T period is a solution of :

$$P(X > u) \times \left(1 + \xi \frac{L_t}{\sigma_u}\right)^{\frac{-1}{\xi}} = \frac{1}{T} \tag{A19}$$

We thus have the GPD equation:

$$L_t = u + \frac{\sigma}{\xi} \times \left(\left(\frac{1}{T \times P(X > u)}\right)^{-\xi} - 1\right) \tag{A20}$$